Coordinating Creditors

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Abstract

The market for developing country sovereign debt has become increasingly competitive. Is this necessarily good for welfare? Or, is there scope for beneficial government intervention to reduce competition, and promote coordination, among creditors? This paper reviews recent theoretical work on the market for developing country sovereign debt that shows that competition can reduce welfare. Further, it argues that while private sector creditor organizations have been successful at coordinating existing creditors in history, government intervention to discourage entry by new creditors may be welfare improving today.

In the past three decades, the market for developing economy sovereign lending has grown increasingly competitive. Advances in telecommunications and the removal of capital market regulations have reduced the costs of doing business. At the same

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time, the shift away from syndicated bank loans towards bonds, illustrated in Figure One, has opened up the market to larger numbers of investors.

In any ordinary market, these developments would be seen as an unquestionably good thing for ordinarily competition promotes the efficient allocation of resources in the economy by forcing firms to price at marginal cost. However, sovereign debt is not traded in an ordinary market as there is no supra-national legal system that can guarantee the enforcement of a contract. And when contracts must be enforced by informal means – such as through a threatened denial of future credit – efficiency *ex ante* is limited to the extent that creditors can coordinate in imposing punishments *ex post*. This begs the question: Is there a role for government policy to encourage cooperation, and discourage competition, in the market for developing country sovereign debt?

Towards an answer to this question, Section I outlines a simple economy in which an developing country borrows internationally to invest in productive domestic projects. I begin by assuming that contracts can be enforced, but are limited in their complexity, and demonstrate that competition in such a world increases efficiency. The intuition is familiar: when contracts are limited, non-competitive firms extract profits at the cost of an inefficient allocation of resources. As a result, an increase in competition limits the ability of firms to extract profits and promotes efficiency. However, when contracts are sufficiently flexible, non-competitive firms can extract profits through price discrimination without any loss of efficiency. And given that international financial markets are increasingly sophisticated, this suggests that the efficiency benefits

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1 Related concerns apply to other informal mechanisms for contract enforcement. For example, if default is deterred by the threat of trade sanctions, as in Jeremy Bulow and Kenneth Rogoff (1989a), cooperation among *trading partners* is necessary to sustain lending by *creditors*. 

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from increased competition may be small. Section II alters the environment to limit the enforceability of contracts and shows that increased competition may decrease the efficiency of resource allocation. Section III then concludes by extracting lessons for policy from both the theory, and from the history of efforts to coordinate international creditors in practice.

I. ENFORCEABLE DEBT

Consider the problem of a small open economy represented by an agent with linear utility. Time lasts forever and in every even numbered year the country has access to a production opportunity which requires foreign capital \( k \), and produces output of

\[
 f(k) = ak - \frac{b}{2}k^2
\]

the following (odd numbered) year. This particular form for the production function ensures that the marginal product of capital (the country’s demand curve) is linear.

The country borrows capital from a group of \( N \) profit maximizing international creditors, where \( N \) is our index of market competitiveness below. The creditors discount the future at the rate \( r \), while the country discounts at rate \( \rho \geq r \). Potential gains from trade are maximized when the country invests the first-best amount

\[
k^{FB} = \frac{a - (1 + r)}{b}
\]

in each even numbered year. To begin, I assume that \( \rho = r \) so that the only motive for trade in capital is for production; as a result, in a first-best world the distribution of odd period surplus

\[
S^{FB} = \frac{[a - (1 + r)]^2}{2b}
\]

is indeterminate both across creditors and the country, as well as through time.
To understand the effects of competition on efficiency in the market for emerging market debt, I begin by assuming that all contracts can be costlessly enforced, but are limited in their complexity. In particular, suppose that in each period creditors offer simple loan contracts at interest rate $r^B$ with the size of the loan determined by the country. I assume that creditors can make offers anonymously (this rules out strategies for the country in which it induces a price war by always breaking ties in favor of one creditor). This game has been designed to mimic a repeated Bertrand oligopoly game: faced with a price, the country chooses quantities off its demand curve, while anonymity ensures symmetric tie-breaking (in expected value).

There are many subgame perfect equilibria of this game, and so I focus on the symmetric one that maximizes cooperation (collusion) by creditors. If creditors cooperate, the best they can achieve for themselves is an equal split of the monopoly level of profits

$$\pi^M = \frac{(a - (1 + r))^2}{4b},$$

which is attained with a quantity of loans half as large as the first best. This produces a level of world welfare three-quarters of the efficient level. But this can be achieved only if the number of creditors is sufficiently small. If a member of the group of creditors were to deviate from this cooperative arrangement, they could capture the entire profits in one period for themselves by slightly undercutting the monopoly price. The worst punishment that can be levied against such a creditor is the threat of reversion of the competitive price $r$ with consequently zero profits. As a result, collusion can be sustained as long as the number of creditors is not too large, or

$$N \leq N^* \equiv \frac{(1 + r)^2}{(1 + r)^2 - 1}.$$  

In this case, the ability of creditors to contract with the country is limited exoge-
nously, and so creditors extract surplus from the country through inefficiently low levels of capital flows. As competition increases, the ability of creditors to act non-competitively is limited and consequently the deviation from the efficient allocation is reduced.

Results like this one lead to the common presumption that competition is good for efficiency. However, the restriction on contract form seems less appealing when applied to international financial markets (which are increasingly sophisticated) and to loans with sovereign governments (where price discrimination seems reasonably easy to sustain).

Instead, suppose that creditors are able to offer contracts that specify pairs of loan amounts and repayments, and as above assume that these offers can be made anonymously. From the perspective of the creditors, the worst subgame perfect equilibrium of this repeated game involves all creditors offering the first best loan amount at interest rate $r$. Given this, it can also be established that, as long as the number of creditors is less than $N^*$, the creditors are able to collude and extract the entire surplus $S^{FB}$ from the country. As before, when $N \leq N^*$, a share of the total surplus in every period is worth more to a creditor than one period of the surplus to itself. For larger $N$, only the competitive outcome is an equilibrium.

The implications for world welfare are very different from the simple loan contract case. Here, independently of the number of creditors, investment is efficient. This result is important for policy: if international financial markets are sufficiently complex so as to allow price discrimination, the welfare gains from greater competition are likely to be small even if enforcement is perfect. It is true that in both models, competition shifts the allocation of welfare in favor of the developing country.
But the redistributive consequences of greater competition can also be duplicated by other policies. This may be particularly important when enforcement is imperfect for, as I show in the next section, in such a world increased competition can decrease efficiency.

II. UNENFORCEABLE DEBT

When contracts are unenforceable, the set of contracts that can be sustained in equilibrium is limited by the necessity that each contract be self-enforcing. I begin by characterizing the set of allocations that can be sustained when contracts are unenforceable, and then demonstrate how they can be implemented as an equilibrium of a game between the country and a single creditor. I then show how the ability to implement this outcome varies as I change the number of creditors, and how results can vary as I change assumptions on the ability of both country and creditor to commit.

To begin, assume that the amount of resources that can be transferred from the developing country to the creditor in any period $T_t$ is constrained by the fact that neither the creditor nor the country can commit to honor a contract. Under this assumption of two-sided limited commitment, the best allocations that can be achieved solve the problem of maximizing the level of welfare provided to the country

$$
\left( \frac{\rho}{1 + \rho} \right) \sum_{s=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^{2s+1} (f(-T_s) - T_{s+1}),
$$

subject to a constraint on creditor profits and sequences of continuing participation constraints for both the creditor and the country.

Inspection of the objective function for the country reveals that by defining the
following time-dependent period utility function

\[
u_s(T) = \begin{cases} 
  f(T) / (1 + \rho) & \text{if } s \text{ is even} \\
  T & \text{if } s \text{ is odd}
\end{cases}
\]

and ignoring non-negativity constraints, the problem can be rewritten as one of maximizing

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t u_t(-T_t),
\]

subject to sequences (one for each \(t\)) of participation constraints for the creditor and the developing country

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{s=t}^{\infty} \left( \frac{1}{1 + \rho} \right)^{s-t} u_s(-T_s) \geq 0,
\]

\[
\left( \frac{r}{1 + r} \right) \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} T_s \geq 0,
\]

and a constraint on initial creditor profits

\[
\left( \frac{r}{1 + r} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t T_t \geq S,
\]

where the parameter \(S\) indexes the distribution of initial surplus. This is an entirely standard limited commitment problem. The constraint set is convex, and the first order conditions for an interior maximum can be rearranged to yield

\[
\left( \frac{1 + r}{1 + \rho} \right)^t u'_t(-T_t) = \lambda + \frac{\sum_{s=0}^{t} \mu_s^C}{1 + \sum_{s=0}^{t} \mu_s^D},
\]

where the \(\mu_i^j\) are current value multipliers on the continuing participation constraints of the Developing country \((i = D)\) and Creditor \((i = C)\) respectively, and \(\lambda\) is the multiplier on the creditor’s profit constraint.

This problem nests a number of interesting cases. For example, if both the country and the creditor can commit to honoring their contracts (so that neither sequence of
participation constraints ever binds) and \( r = \rho \) (so that the economy is at an interior solution with respect to consumption) the first-order conditions can be rearranged to give the first best level of investment in every even period, or \( f'(-T_t) = 1 + r \).

Next, suppose that the country cannot commit to honoring its contracts, but that creditors are able to use the legal systems of the developed economies in which they are based to commit themselves. This case of one-sided limited commitment corresponds to ignoring the creditor’s continuing participation constraint in the above programming problem, or setting \( \mu^C_t = 0 \) for all \( t \). Now the first order condition gives

\[
\left( \frac{1 + r}{1 + \rho} \right)^t u_t'(-T_t) = \frac{\lambda}{1 + \sum_{s=0}^{t} \mu_s^D}.
\]

If \( \rho = r \), the marginal utility of consumption for the country is non-increasing, eventually converging to a constant where the participation constraint of the country never binds. The reason is intuitive: as the creditor can commit to repaying contracts, the country responds by accumulating a buffer-stock of savings with the creditor which it uses to finance its investments over time. That is, the country becomes a net-saver in international markets.

In order to studying borrowing, assume that \( \rho > r \). If \( t \) is odd, this gives

\[
\left( \frac{1 + r}{1 + \rho} \right)^t = \frac{\lambda}{1 + \sum_{s=0}^{t} \mu_s^D},
\]

so that in period \( t + 1 \) (even) we have

\[
\left( \frac{1 + r}{1 + \rho} \right)^{t+1} \left( \frac{1}{1 + \rho} \right) f'(-T_{t+1}) = \frac{\lambda}{1 + \sum_{s=0}^{t} \mu_s^D} \frac{1 + \sum_{s=0}^{t} \mu_s^D}{1 + \sum_{s=0}^{t+1} \mu_s^D}
\]

and hence

\[
\frac{1 + r}{(1 + \rho)^2} f'(-T_{t+1}) = 1,
\]
because $\mu_{t+1}^D = 0$. That is, the country invests less than the first best investment level. Together with the participation constraints of the country, this equation pins down the constrained efficient allocation for this economy. All efficient allocations have a similar form: after an initial loan that varies in size according to the distribution of surplus, the economy converges to a stationary equilibrium in which the constrained efficient amount is invested.

In Wright (2004a), I show how to implement this allocation as the equilibrium of a game in which a creditor makes contract offers to a country so that the creditor has all the bargaining power. The creditor is assumed to have access to a commitment technology for delivering on promises next period. This technology allows the creditor to both take deposits, and to make loan commitments, where the latter serve to bind the creditor not to extract too much surplus in the future. Given an initial loan commitment, this game has a unique subgame perfect equilibrium which is constrained efficient.

When the number of creditors in the market increases, the potential for multiple equilibria arises. The worst equilibrium involves creditors making zero profits by offering deposit contracts at interest rate $r$. As long as the number of creditors is not too large, the threat of reversion to this competitive deposit-taking equilibrium can support the constrained efficient level of capital flows. To see this, note that a deviating creditor can offer a range of contracts that include both deposit and loan components. If this creditor offers a loan, the country will default on the loan next period and deposit the proceeds with the other creditors (by an argument familiar from Bulow and Rogoff 1989b). But if this creditor offers a deposit contract, it can induce the country to default on its other loans and make a profit. When an individual
creditor’s share of the profits from lending is small, and they are not too impatient, this becomes more profitable than lending, and hence no lending can be sustained in equilibrium.

Unlike the simple loan case studied above, the “competitive” outcome in this case is not constrained efficient: investment in the country is in general too low, at least in the initial period. That is, when enforcement is limited, competition in the market for developing country sovereign debt reduces efficiency. Importantly, competition is defined by the number of creditors and not by the profitability of those creditors. In fact, and also unlike the simple loan contract model studied above, cooperation can be sustained even when creditors make zero-profits (so that the market may appear very competitive on the surface). The reason lies in the inherently inter-temporal nature of the loan contract: even if expected repayments leave all creditors indifferent to making the loan, threats of disrupting future repayments support cooperation in equilibrium.

Perhaps a little surprisingly, when neither country nor creditor can commit (so that in general both sequences of participation constraints may bind) capital flows may be more efficient as the ability of creditors to coordinate in imposing punishments on the country is improved: deviations that involve deposit contracts are no longer feasible. Narayana Kocherlakota (1996) has shown how the constrained efficient allocation with two-sided limited commitment can be implemented as one of the subgame perfect equilibria of a game between a country and a creditor in which each party makes simultaneous transfers to each other at the start of every period. This result carries over to the present environment.

To see the effect of increases in competition in this game, imagine that the number of creditors playing this game is increased. If the original equilibrium strategy profile
is modified to include zero transfers to these new creditors, which are reciprocated by zero transfers in turn, it is easy to see that this adjusted strategy profile implements the same allocation. That is, when both the country and creditors cannot commit to honoring contracts, changes in the number of competing creditors have no effect on the efficiency of the market for developing country sovereign debt. Nor is the result solely an artifact of the multiplicity of equilibria of this simultaneous move game; Kenneth M. Kletzer and Brian D. Wright (2000) drastically refine the set of equilibria by insisting on coalition proofness for their economy and show that the set of coalition proof equilibria is also invariant to the number of competing creditors.

**III. POLICY LESSONS**

Is there room for beneficial intervention by creditor country governments in the market for developing country sovereign debt? If so, what form should intervention take? Perhaps counter-intuitively, the theory warns against direct attempts to improve contract enforcement. This is because national sovereignty inevitably limits the ability to enforce contracts with developing country governments, so that enforcement mechanisms have their largest effect on creditors. And when enforcement is asymmetric, improvements in enforcement that expand potential gains from trade *ex ante* may be offset by reductions in the ability to cooperate *ex post* (this assumes that enforcement cannot be tailored to prevent fraudulent conveyancing as in Philip Bond and Arvind Krishnamurthy 2004).

The theory also demonstrates that *if* barriers to entry limit the number of creditors and *if* these creditors cooperate effectively, the constrained efficient allocation of resources is attainable without government intervention. Consequently, if these con-
ditions are absent there may be room for beneficial government intervention. History gives us reason to be optimistic about the latter condition: private sector efforts to coordinate market participants appear to have been successful in practice. In Wright (2004b), I trace the formation of one of these institutions – the British Council of Foreign Bondholders – to an 1860’s violation of a creditor embargo of Spain, and argue that the Council and its successor Corporation were effective in deterring further embargo violations (see Paulo Mauro and Yishay Yafeh 2003 for an alternate interpretation). However, efficiency also requires that existing creditors coordinate with potential new entrants and, over time, barriers to entry into the market for developing country sovereign debt have fallen. This suggests that government intervention in the form of restrictions on entry may be necessary for efficient outcomes.
REFERENCES


Figure 1: Developing Country Sovereign Debt
From Private Sector Creditors (Percentage of Gross National Income)
TECHNICAL APPENDIX

Repeated Bertrand Oligopoly

Under our anonymity assumption, if there are \( N \) firms, and the developing country allocates quantities according to the proportions \( \{p_n\}_{n=1}^{N} \), with \( \sum_{n=1}^{N} p_n = 1 \) (which could also be interpreted as expected shares given random allocations with probabilities \( p_n \)) each firm gets

\[
\sum_{n=1}^{N} \frac{p_n}{N} = \frac{1}{N},
\]

which is equivalent to a symmetric tie breaking rule. This anonymity assumption is important because it rules out strategies for the country in which it always splits ties in favor of one creditor, thus inducing a price war by giving other creditors a strict preference to undercut.

Under this rule, it is immediate that the competitive price attains the worst (from the perspective of the creditors) subgame perfect Nash equilibrium payoffs of zero. In turn these payoffs sustain the monopoly price, with each creditor receiving an equal share (in expected value) of monopoly profits as long as

\[
\frac{\pi^M}{N} \geq \left( 1 - \left( \frac{1}{1+r} \right)^2 \right) \pi^M,
\]

or

\[
N \leq N^* \equiv \frac{(1 + r)^2}{(1 + r)^2 - 1}.
\]

Repeated Cournot Oligopoly a la Cronshaw and Luenberger

To map our problem into that of a Cournot oligopoly game, we assume that creditors make offers of loan quantities. To determine prices, we postulate the existence
of a Cournot auctioneer who takes quantity offers and determines a price from the country’s demand curve for capital, and then offer the combined bundle of prices and quantities to the developing country. With prices determined from the demand curve, the country always accepts the offer.\(^2\)

This environment is identical to a repeated Cournot oligopoly model. To see how this can be solved in closed form using the Cronshaw-Luenberger algorithm, note first that the demand curve is linear and of the form

\[ P = a - bK, \]

and costs are \(1 + r\) per unit. We write the model net of costs (which is equivalent to redefining \(a\) as the difference between \(a\) and costs \(1 + r\)).

Profits per creditor if all \(n\) creditors play \(k\) are

\[ \pi(k) = (a - bnk)k, \]

so that the best a creditor can do if everyone else plays \(k\) is given by

\[ \pi^*(k) = \max_{k'} (a - b[(n - 1)k + k'])k', \]

which has first order condition

\[ a - b(n - 1)k + 2bk' = 0, \]

or

\[ k' = \frac{a - b(n - 1)k}{2b}, \]

\(^2\)We conjecture, following Kreps and Scheinkman (1983), that a two-stage process in which creditors choose quantities and then offer contracts in prices, would produce the same outcomes.
so that
\[
\pi^* (k) = \left( a - \left[ b (n - 1) k + \frac{a - b (n - 1) k^3}{2} \right] \right) \frac{a - b (n - 1) k}{2b} \\
= \frac{(a - b (n - 1) k)^2}{4b}.
\]

Therefore, the difference between these two functions is
\[
\Delta (k) \equiv \pi^* (k) - \pi (k) \\
= \frac{(a - b [(n - 1) k])^2}{4b} - (a - b nk) k \\
= \frac{(a - (n + 1) bk)^2}{4b}.
\]

The best subgame perfect equilibrium (with action \( \bar{k} \) and value \( \bar{w} \)) of a repeated game is obviously self-rewarding. It must also be enforceable given the threat of reversion to the worst subgame perfect equilibrium (with value \( w \)). Hence it must be that
\[
(1 - \delta) \pi (\bar{k}) + \delta \bar{w} \geq (1 - \delta) \pi^* (\bar{k}) + \delta w,
\]
which can be rearranged to get
\[
\delta (\bar{w} - w) \geq (1 - \delta) \Delta (\bar{k}).
\]

On the other hand, the worst equilibrium of a repeated game (with action \( k \) and value \( w \)) is obviously self-enforcing, and must be rewarded by a continuation value no better than the best equilibrium, so we have
\[
(1 - \delta) \pi (k) + \delta w \geq (1 - \delta) \pi^* (k) + \delta \bar{w},
\]
which can be rearranged to get
\[
\delta (\bar{w} - w) \geq (1 - \delta) \Delta (k).
\]
This gives rise to the following algorithm for finding the largest level of “deterrence” \( D^* \equiv \bar{w} - \underline{w} \). Suppose that the maximum deterrence level of the game is \( D \). Then define two functions

\[
\begin{align*}
    f (D) &= \max_k \{ \pi (k) : \Delta (k) \leq D \}, \\
    g (D) &= \min_k \{ \pi^* (k) : \Delta (k) \leq D \}.
\end{align*}
\]

We can then solve for the largest \( D \) such that

\[
D = \frac{\delta}{1 - \delta} [f (D) - g (D)].
\]

In our context, how do we find \( f \)? If \( D \) is such that the constraint does not bind, clearly the optimal choice of quantities is a share of the monopoly level

\[
a \frac{2b}{n}.
\]

which gives profits

\[
a^2 \frac{4b}{4bn}.
\]

This will be true if deterrence is such that

\[
\frac{a^2 \left( \frac{n+1}{2n} \right)^2}{4b} \leq D.
\]

If the constraint binds, then \( k \) must satisfy

\[
k = \frac{a \pm 2\sqrt{bD}}{(n + 1) b},
\]

so that profits are either

\[
\frac{(a + n2\sqrt{bD}) (a - 2\sqrt{bD})}{b (n + 1)^2}
\]

\[
= \frac{a^2 - 4nbD + 2a\sqrt{bD} (n - 1)}{b (n + 1)^2}
\]
for the negative root, or in the case of the positive root
\[
\frac{a^2 - 4nbD - 2a\sqrt{bD}(n - 1)}{b(n + 1)^2},
\]
which is in general smaller and so does not maximize profits. As we require \( k \geq 0 \), we also have to check that
\[
k_- = \frac{a - 2\sqrt{bD}}{(n + 1)b} > 0,
\]
which requires only that
\[
D < \frac{a^2}{4b}.
\]
But if the constraint binds, we have
\[
D \leq \frac{a^2 \left(\frac{n-1}{2n}\right)^2}{4b} < \frac{a^2}{4b},
\]
and so we do not have to worry about this case. That is
\[
f(D) = \begin{cases} 
\frac{a^2}{4bn} & \text{if } \frac{a^2 \left(\frac{n-1}{2n}\right)^2}{4b} \leq D_f \leq D \\
\frac{a^2 - 4nbD - 2a\sqrt{bD}(n - 1)}{b(n+1)^2} & \text{if } 0 \leq D \leq \frac{a^2 \left(\frac{n-1}{2n}\right)^2}{4b}
\end{cases}.
\]

As for \( g \), if constraint does not bind, we get
\[
k = \frac{a}{b(n - 1)},
\]
which implies \( \pi^* = 0 \). This works if
\[
\frac{a^2 \left(\frac{1}{(n-1)}\right)^2}{b} \leq D.
\]
If it does bind, then \( k \) is as before and we get either
\[
\pi^* = \frac{a^2 + (n - 1)^2 bD + a (n - 1) 2\sqrt{bD}}{b(n + 1)^2}
\]
with the negative root, or

$$\pi^* = \frac{a^2 + (n - 1)^2 b D - a (n - 1) 2 \sqrt{b D}}{b (n + 1)^2}$$

with the positive root. The second is always smaller, and so we have

$$g(D) = \begin{cases} 
0 & \text{if } \frac{a^2 \left(\frac{1}{n-1}\right)^2}{b} \equiv D_g \leq D \\
\frac{a^2 + (n-1)^2 b D - a(n-1) 2 \sqrt{b D}}{b(n+1)^2} & \text{if } 0 \leq D \leq \frac{a^2 \left(\frac{1}{n-1}\right)^2}{b}.
\end{cases}$$

It remains to find the largest fixed point \(D\) of

$$D = \frac{\delta}{1 - \delta} [f(D) - g(D)].$$

Note that \(D = 0\) is a fixed point for all \(\delta\). It is clear that \(g(D)\) is strictly convex in \(D\) on the second piece (low \(D\)), and constant thereafter. Similarly, \(f(D)\) is strictly concave for low \(D\), and constant for high \(D\). That is, \(f - g\) is strictly concave or constant, and therefore has a unique maximum in \(D\).

It is useful to divide the possibilities up into a number of regions. If the parameters are such that the largest fixed point occurs for when both functions are constant, then it must be that we are in the region defined by

$$D \geq \max \left\{ D_f \equiv \frac{a^2 \left(\frac{n-1}{2n}\right)^2}{4b}, D_g \equiv \frac{a^2 \left(\frac{1}{n-1}\right)^2}{b} \right\}.$$ 

If \(N \geq 6\), then this region is defined by \(D_f\) which holds for \(\delta \geq \delta_f^*\) which solves

$$\frac{1 - \delta_f^* a^2 \left(\frac{n-1}{2n}\right)^2}{\delta_f^* a^2 / 4b} = \frac{a^2}{4bn}$$

which can be rearranged to get

$$\delta_f^* = \left(\frac{n-1}{n+1}\right)^2.$$
If $N \leq 5$, the region is defined by $D_g$ which holds for $\delta \geq \delta_g^*$ which solves

$$1 - \frac{\delta^*}{\delta_g} \frac{a^2}{b} \left(\frac{1}{n-1}\right)^2 = \frac{a^2}{4bn}$$

which can be rearranged to get

$$\delta_g^* = \frac{4n}{(n+1)^2}.$$ 

If $\delta$ is very low (so that $D \leq \min \{D_f, D_g\}$), then $D$ and $\delta$ are related by

$$1 - \frac{\delta}{\delta} D = \frac{a^2 - 4nbD + 2a\sqrt{bD} (n-1)}{b (n+1)^2} - \frac{a^2 + (n-1)^2 bD - a (n-1) 2\sqrt{bD}}{b (n+1)^2}$$

which can be rearranged to get

$$D = \left(\frac{\delta 4a (n-1)}{b (n+1)^2}\right)^2 b.$$

This can then be substituted back into $f(D)$ to get the maximum level of profits.

For intermediate levels of $\delta$, we can begin by solving the case where $N \geq 6$. In this case, we have

$$1 - \frac{\delta}{\delta} D = \frac{a^2 - 4nbD + 2a\sqrt{bD} (n-1)}{b (n+1)^2}$$

If $N \leq 5$, the equivalent expression is

$$1 - \frac{\delta}{\delta} D = \frac{a^2}{4bn} - \frac{a^2 + (n-1)^2 bD - a (n-1) 2\sqrt{bD}}{b (n+1)^2}.$$ 

These can be solved for $D$ which in turn imply maximum sustainable profits.

**Repeated Price Discriminating Oligopolist**

The action space for each stage game consists of two numbers for each creditor that may be thought of as an initial loan amount $k$ and a repayment amount $T$. Suppose
that the set of creditors announce pairs \( \{(k_n, T_n)\}_{n=1}^N \). As before, we assume that creditors can make offers anonymously and that the country chooses either convex combinations of these contracts or equivalently probabilities of accepting a contract, to maximize its own surplus. This gives us the analog of a symmetric tie-breaking rule.

Under these assumptions we can show that the strategy profile specifying the competitive loan contract \((k^{FB}, (1 + r)k^{FB})\) for all creditors in all periods attains the worst subgame perfect equilibrium payoff for the creditors. To see that this is a subgame perfect strategy profile, note that any deviations that offer loans on better terms are accepted and make negative profits, while loans on worse terms are rejected. The country never rejects the competitive offer. This profile achieves a zero payoff for all creditors. To see that it is the worst, note that independently of the strategies played by all other creditors, a creditor can guarantee itself a payoff of zero by offering this contract.

Given this result, we can prove the following for symmetric strategies among creditors: if \( N \leq N^* \), the best strongly symmetric subgame perfect equilibrium value is attained by the profile \((k^{FB}, S^{FB})\) supported by reversion to the competitive loan contract. If \( N > N^* \), the unique strongly symmetric subgame perfect equilibrium profile is the repeated competitive loan contract.

The way to see this is to note that if every creditor offers the collusive profile \((k^{FB}, S^{FB})\), each receives a one-\(N\)'th share in expected value of these profits in each period. The best any deviating creditor can achieve is to offer the first best loan and to undercut the surplus payment by an arbitrarily small amount. Such a deviation produces a one-time payoff of \(S^{FB}\), followed by zero profits thereafter. But this is not
optimal for the deviating creditor if \( N \leq N^* \) by definition of \( N^* \). If \( N > N^* \), a similar argument shows that there exists profitable deviations from any strongly symmetric profile other than the competitive loan contract.

**TWO SIDED LIMITED COMMITMENT**

As described in the text the country values consumption only in odd periods, so that given a sequences of transfers \( \{T_t\} \) from the country to the agent (which can be negative) they would be valued at

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{s=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^{2s} (f (-T_s) - T_{s+1}).
\]

Inspection of the objective function for the country reveals that if we define the following time-dependent period utility function

\[
u_s(T) = \begin{cases} 
\beta f(T) & \text{if } s \text{ is even} \\
T & \text{if } s \text{ is odd} 
\end{cases}
\]

we can rewrite the welfare of the country as

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^{t} u_t (-T_t).
\]

The problem of finding optimal allocations subject to two-sided limited commitment then becomes one of maximizing

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^{t} u_t (-T_t),
\]

subject to a constraint on creditor profits

\[
\left( \frac{r}{1 + r} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^{t} T_t \geq S,
\]
where as we vary this parameter $S$, we can trace out the entire frontier of efficient self-enforcing allocations, as well as sequences (one for each $t$) of participation constraints for the creditor and the country

$$
\left(\frac{\rho}{1+\rho}\right) \sum_{s=t}^{\infty} \beta^{s-t} u_s (-T_s) \geq 0,
$$

$$
\left(\frac{r}{1+r}\right) \sum_{s=t}^{\infty} q^{s-t} T_s \geq 0.
$$

That is, by rewriting the problem in this way, we have transformed the limited commitment problem with production into one that involves consumption smoothing in response to alternating tastes/utilities.

This is a convex programming problem, and the first order conditions for an interior maximum can be rearranged to get

$$
\left(\frac{1+r}{1+\rho}\right)^{t} u'_t (T_t) = \frac{\lambda + \sum_{s=0}^{t} \mu^C_s}{1 + \sum_{s=0}^{t} \mu^D_s},
$$

where the $\mu^i_t$ are current value multipliers on the continuing participation constraints of the developing country ($i = D$) and creditor ($i = C$) respectively, and $\lambda$ is the multiplier on the creditors profit constraint. Note that if both countries can commit to honoring their contracts, so that neither sequence of participation constraints ever binds, and

$$
q \equiv \frac{1}{1+r} = \beta \equiv \frac{1}{1+\rho},
$$

(so that we are at an interior solution with respect to consumption) this can be rearranged to give

$$
q f'(T_t) = 1,
$$

for all $t$ even, which yields the first best level of investment.
In the general case where neither country can commit, so that in general both sequences of constraint may bind, we assume $\rho = r$. If $t$ is odd, we know that $u^t_t = 1$ so that

$$1 = \frac{\lambda + \sum_{s=0}^{t} \mu_s^B}{1 + \sum_{s=0}^{t} \mu_s^C}.$$

Therefore, in period $t + 1$ we have

$$q^t f^t (T_{t+1}) = \frac{\lambda + \sum_{s=0}^{t+1} \mu_s^B}{1 + \sum_{s=0}^{t+1} \mu_s^C},$$

which can be rearranged to get

$$q^t f^t (T_{t+1}) = \frac{\lambda + \sum_{s=0}^{t} \mu_s^B}{1 + \sum_{s=0}^{t} \mu_s^C} \frac{1}{1 + \sum_{s=0}^{t+1} \mu_s^C}.$$

In period $t + 1$, the country receives a transfer from the creditor so that $\mu_{t+1}^C = 0$, whereas $\mu_{t+1}^B > 0$, so that using the condition for period $t$ we get

$$q^t f^t (T_{t+1}) = \frac{\lambda + \sum_{s=0}^{t+1} \mu_s^B}{1 + \sum_{s=0}^{t+1} \mu_s^C} = \frac{\lambda + \sum_{s=0}^{t+1} \mu_s^B}{\lambda + \sum_{s=0}^{t} \mu_s^B} > 1,$$

which implies that we have less investment than in the first best.

We focus upon stationary equilibria which are described by two numbers: $k$, an amount transferred to the country in even periods, and $T$, an amount transferred to the creditors in odd periods. It is straightforward to see that these numbers can be determined from

$$T^{TS} = \left( \frac{1}{1+r} \right)^2 f_k^{TS},$$

$$T^{TS} = (1+r) k^{TS}.$$

In related environments, Narayana Kocherlakota (1996) and Kenneth M. Kletzer and Brian D. Wright (2000) have shown how this allocation can be implemented as one of the subgame perfect equilibria of a game between a country and a creditor.
in which each party makes simultaneous transfers to each other at the start of every period. This result carries over to the present environment.

To see that this is true here, consider a game where in each period both the developing country and the creditor simultaneously make non-zero transfers to each other. It is easy to see that one subgame perfect equilibrium of this game is autarky: if both players transfer zero in each period, no player can gain by deviating and making a positive transfer. It is also easy to see that this is the worst subgame perfect equilibrium: a player can always guarantee a payoff no worse than zero (the autarkic level) by transferring zero in every period. Then, using the well known results of Abreu (1988) we can characterize the full set of subgame perfect equilibria by threat of reversion to autarky. The frontier is found from the programming problem studied above.

The strategies that support the allocation in the two player game can be found by computing the net transfer between agents from the solution to the planning problem. The strategies then have the relevant player transfer this amount as long as the other player has made their transfer in all previous periods. If not, both players transfer zero thereafter.

**ONE SIDED LIMITED COMMITMENT**

Somewhat surprisingly, modest changes to the contracting environment can produce radically different results. For example, suppose that creditors are able to use the legal systems in developed economies to commit themselves to honoring contracts, whereas developing countries cannot commit. This case of *one-sided limited commitment* corresponds to ignoring the creditors continuing participation constraint in the
above programming problem, or setting $\mu^B_t = 0$ for all $t$. Now the first order condition gives

$$(\frac{1 + r}{1 + \rho})^t u'_t(T_t) = \frac{\lambda}{1 + \sum_{s=0}^t \mu^C_s}.$$ 

If $\rho = r$, the marginal utility of consumption for the country is non-decreasing, eventually converging to a constant where the participation constraint of the country never binds. The reason is intuitive: as the creditor can commit to repaying contracts, the country responds by accumulating a buffer-stock of savings with the creditor which it uses to finance its investments over time. That is, the country becomes a net-saver in international markets.

In order to studying borrowing, we assume that $\rho > r$. If $t$ is odd, this gives

$$(\frac{1 + r}{1 + \rho})^t = \frac{\lambda}{1 + \sum_{s=0}^t \mu^D_s},$$

so that in period $t + 1$ we have

$$(\frac{1 + r}{1 + \rho})^{t+1} \beta f' (T_{t+1}) = \frac{\lambda}{1 + \sum_{s=0}^{t+1} \mu^D_s} \frac{1 + \sum_{s=0}^t \mu^D_s}{1 + \sum_{s=0}^{t+1} \mu^D_s}$$

and hence

$$\frac{1 + r}{(1 + \rho)^2} f' (T_{t+1}) = 1,$$

because $\mu^D_{t+1} = 0$, and we have less than the first best investment level. Together with the participation constraint of the country, this equation pins down the stationary allocation in this economy

$$k^{OS} = f^{-1} \left( \frac{(1 + \rho)^2}{1 + r} \right),$$

$$T^{OS} = \frac{f \left( k^{OS} \right)}{(1 + \rho)^2}.$$
Given our specification for $f$ the first equation gives

$$k_{OS} = \frac{a - \frac{(1+\rho)^2}{1+r}}{b},$$

while the second implies

$$T_{OS} = \frac{a - \frac{(1+\rho)^2}{1+r}}{b} - \frac{b}{2} \left( \frac{a - \frac{(1+\rho)^2}{1+r}}{b} \right)^2 \frac{(1+\rho)^2}{1+r}$$

$$= \frac{a^2 - a(1+\rho)^2}{b} - \frac{a^2 + (1+\rho)^4}{2b} - \frac{2a(1+\rho)^2}{2b}$$

$$= \frac{(a - \frac{(1+\rho)^2}{1+r}) (a + \frac{(1+\rho)^2}{1+r})}{2b (1+\rho)^2}.$$  

Note that, from the fact that the participation constraint of the country bounds in odd periods, we have that the value they receive as part of the stationary allocation is given by $V_o^D = 0$, while in even periods the stationary value is

$$V_e^D = \frac{\rho}{(1+\rho)^2} f(k_{OS}).$$

The creditor, on the other hand, receives

$$\Pi_e^C = \frac{r}{1+r} \left[ T_e + \frac{1}{1+r} T_o + \left( \frac{1}{1+r} \right)^2 T_e + \left( \frac{1}{1+r} \right)^3 T_o + \ldots \right]$$

$$= \frac{r}{1+r} T_e + \frac{r}{1+r} T_o$$

$$\Pi_o^C = \frac{r}{1+r} T_o + \frac{1}{1+r} T_e.$$  

Substituting for $T_o$ gives

$$\Pi_e^C = \frac{(1+r) T_e + \frac{1}{(1+\rho)^2} f(-T_e)}{2+r},$$

$$\Pi_o^C = \frac{\frac{1+r}{(1+\rho)^2} f(-T_e) + T_e}{2+r}.  $$
As a result of our linear-quadratic assumptions, the stationary allocation provides an almost complete characterization of the solution to the one sided limited commitment problem in this case. To compute allocations in the first periods, suppose to begin that the participation constraint of the country does not bind in period zero, but does bind in period one. Then using the first order conditions and $\mu_0^D = 0$ and $\mu_1^D > 0$ we have
\[
\frac{1}{1 + \rho} f'(T_0) = \lambda,
\]
which determined initial period capital flows as a function of the distribution of surplus, parameterized by $\lambda$. In period one, as the constraint binds we get
\[
-\frac{\rho}{1 + \rho} T_1 + \frac{\rho}{1 + \rho} V_e^D = 0,
\]
where $V_e^D$ is the value function from period two onwards, reflecting the fact that the country will be in the stationary distribution. But substituting for this we get
\[
T_1 = \frac{1}{(1 + \rho)^2} f(k^{OS}),
\]
which is to say that the economy converges to the stationary distribution after one period.

As a result of the country getting zero in period one, the country’s welfare in period zero is given by
\[
\frac{1}{(1 + \rho)^2} f(-T_0),
\]
while the creditors profits are
\[
\Pi_0^C = \frac{r}{1 + r} T_0 + \frac{1}{1 + r} \Pi_o^C,
\]
so that $T_0$ is chosen from this according to the initial surplus distribution parameterized by $S$. 29
Suppose that the country’s participation constraint binds in the first period, but not in the second. Using $\mu_0^D > 0$ and $\mu_1^D = 0$ we get

$$\frac{1}{1 + \rho} f' (-T_0) = \frac{\lambda}{1 + \mu_0^D} = \frac{1 + r}{1 + \rho},$$

which implies $-T_0 > 0$, a contradiction of the participation constraint binding in the first period.

Suppose that the country’s participation constraints bind in both periods. Then the country gets no transfer in the first period, and the stationary allocation begins in the second period (period one). This corresponds to the first case considered for the largest $S$.

Finally, note that there exist solutions, parameterized by low (perhaps negative) $S$ in which the country’s participation constraints do not bind for the first two periods before the stationary allocation begins in the third period.

**Implementation**

In this subsection, we sketch the proof of implementation.

Consider a game in which the creditor is able to make binding promises of payment the next period, conditional upon the actions of the country today. At the start of each period $t$, any past promised payments $P_t$ are made, and the creditor “offers to the agent” or commits to a function $P_{t+1}$ which specifies promises made for tomorrow. In addition to the past history of play, this function is conditioned upon actions by the country – transfers to the creditor $T_t$ – later in period $t$.

After observing the creditors action, the country then makes a transfer. We typically consider strategies for the creditor in which they promise a positive transfer tomorrow conditional upon a specific transfer from the country today. If the country
makes this transfer we will say that they have “accepted” the offer.

The proof of implementation follows from four steps. First, that note that the efficient allocations solve the problem of maximizing creditor profits

\[
\left( \frac{r}{1 + r} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t T_t,
\]

subject to a constraint on country utility

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t u_t (-T_t) \geq S,
\]

and sequences (one for each \( t \)) of participation constraints for the country

\[
\left( \frac{\rho}{1 + \rho} \right) \sum_{s=t}^{\infty} \beta^{s-t} u_s (-T_s) \geq 0,
\]

(which follows from the original statement of the problem and our assumptions on the primitives).

Second, following Worrall (1990), the solution to this problem is equivalent to the solution of the functional equation problem

\[
\Pi (P, j) = \max \left( \frac{r}{1 + r} \right) (T - P) + \left( \frac{1}{1 + r} \right) \Pi (P', j'),
\]

subject to

\[
\left( \frac{\rho}{1 + \rho} \right) u_j (P - T) + \left( \frac{1}{1 + \rho} \right) P' \geq P,
\]

for

\[
P_0 = \frac{1 + \rho}{\rho} S_0,
\]

and where \( j \) indexes whether the period is odd or even, and evolves in the obvious way.

Third, note that the solution to this problem obviously defines strategies that constitute a subgame perfect equilibrium combined with the strategy for the country...
that is to “accept” whenever they are at least weakly indifferent. The creditor cannot do better (as the actions maximize profits), and the country is at least indifferent.

Fourth, and finally, it is necessary to show that all subgame perfect equilibria implement this allocation, which requires that we show that the country never breaks an indifference relation by refusing an offer. To see this suppose not. But as the countries payoff is strictly increasing in its consumption over the relevant range, and as the creditor is strictly better off with the country accepting the offer, the creditor would be strictly better off deviating and offering $\varepsilon$ more to the country in this period, keeping the rest of the strategy unchanged.

**Best Deposit Contracts**

As creditors have access to a commitment technology, they are able to offer deposit contracts. The best deposit contract they can offer, from the perspective of the country, subject to not making losses, is a zero profit deposit contract paying rate $r$. This contract satisfies

$$\max_{C,S} \left( \frac{\rho}{1 + \rho} \right) \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t u_t (C_t),$$

subject to

$$C_t + S_t \leq (1 + r) S_{t-1},$$

and

$$S_t \geq 0,$$

for all $t$ with $S_0$ given. Note that we do not require consumption to be non-negative.

Letting $\lambda_t$ be the Lagrange multipliers on the consumers flow budget constraints,
the first order necessary conditions for an optimum include

\[ \beta^t u_t (C_t) = \lambda_t, \]

\[ \lambda_t \geq \lambda_{t+1} (1 + r). \]

Intuitively, because the consumer can save but not borrow, the value of consumption today is always at least as large as the value of saving for tomorrow.

The stationary equilibrium for this economy is particularly simple with the country saving in odd periods in order to take advantage of the high marginal utility of consumption in even periods. From the budget constraint we have

\[ C_e = (1 + r) S_o \]
\[ C_o = -S_o \]
\[ = -\frac{C_e}{1 + r}, \]

while from the first order conditions we get

\[ 1 = \frac{1 + r}{(1 + \rho)^2} f''(C_e). \]

But this implies that

\[ C_e = -T_e, \]
\[ C_o = \frac{T_e}{1 + r}. \]

Note that this implies the same stationary level of investment (consumption) as the one-sided limited commitment model, so that any inefficiencies concerning the deposit equilibrium can only occur along the transition.
The value to being in the stationary equilibrium of this economy is

\[ V_e^{\text{deposit}} = \frac{\rho}{1 + \rho} \left[ \frac{1}{1 + \rho} f(C_e) + \frac{1}{1 + \rho} C_0 + \left( \frac{1}{1 + \rho} \right)^3 f(C_e) + \left( \frac{1}{1 + \rho} \right)^3 C_0 + \ldots \right] \]

\[ = \frac{\rho}{1 - \left( \frac{1}{1 + \rho} \right)^2} \left[ f(C_e) + C_0 \right] \]

\[ = \frac{f(-T_e) + \frac{1}{1 + r} T_e}{2 + \rho}, \]

\[ V_o^{\text{deposit}} = \frac{\rho}{1 + \rho} \left[ C_0 + \left( \frac{1}{1 + \rho} \right)^2 f(C_e) + \left( \frac{1}{1 + \rho} \right)^2 C_0 + \left( \frac{1}{1 + \rho} \right)^4 f(C_e) + \ldots \right] \]

\[ = \frac{\rho}{1 - \left( \frac{1}{1 + \rho} \right)^2} \left[ C_0 + \left( \frac{1}{1 + \rho} \right)^2 f(C_e) \right] \]

\[ = \frac{1 + \rho}{2 + \rho} \left[ \frac{1}{1 + r} T_e + \left( \frac{1}{1 + \rho} \right)^2 f(-T_e) \right]. \]

**Increases in Competition**

In the following, we sketch the details for the game with multiple creditors. The complete argument for a slightly different environment is in Wright (2004a).

Suppose that we vary the above game by introducing \( N \) creditors who simultaneously make offers to the country. There are multiple subgame perfect equilibria of this game. The worst involves the creditors offering only deposit contracts of a particular type.

Then it is straightforward to show that the strategy profile in which all creditors offer at date \( t \) the payment \((1 + r) S_t^*\) in return for the transfer of \( S_t^*\), and the country accepts one creditors offer today attains the worst subgame perfect equilibrium value. It attains the worst because it yields zero profits, and every creditor can guarantee themselves a zero profit by playing this strategy. It is subgame perfect because any
deviation by a creditor either involves losses (if the deviation involves a loan or a more generous deposit contract) or would not be accepted, while any deviation by the country lowers its welfare.

The threat of deviation to this worst punishment can then be used to identify the best equilibria that can be sustained, and consequently the cases in which the efficient allocations can be sustained by strategies for the creditors in which they each offer $1/N'th$ of the efficient contract. Recall that the efficient allocations generate payoffs of $\Pi(P_t, j)$ in the subgame that begins with the country entering time $t$ with promised payments $P_t$. If a creditor deviates, they cannot escape their obligation for $P_t/N$. They may offer an alternative loan commitment or deposit contract. However, as the remaining creditors will punish a deviation by offering the best possible deposit contract, any loan made by the deviating creditor will never be repaid by the same argument as in Bulow and Rogoff (1989b). This is not profitable. The only potential profitable deviations for the creditor involve offering a deposit contract (which makes profits in the first period) followed by the optimal deposit contract thereafter.

For the case of our specific functional forms, a creditor gets

$$\frac{1+r}{(1+\rho)^2} f(-T_e) + T_e \frac{N}{(2+r)},$$

in odd periods if they stay in the arrangement.

If they defect, the country will pay them the total surplus from the deposit contract in such a period, and the bank will be responsible for offering the deposit contract. As the payment is in the current period, the most a country will pay is

$$\frac{1+\rho}{\rho} V_{o depot} = \frac{1+\rho}{\rho} \frac{1 + \rho}{2 + \rho} \left[ \frac{1}{1+r} T_e + \left( \frac{1}{1+\rho} \right)^2 f(-T_e) \right].$$

Of course, next period the deviating creditor is liable for $-T_e$. Therefore, cooperation
can be sustained as long as

\[
\frac{1+r}{(1+\rho)^2} f(-T_e) + T_e \geq \frac{r}{1+r \rho (2+\rho)} \left[ \frac{1}{1+r} T_e + \left( \frac{1}{1+\rho} \right)^2 f(-T_e) \right] + \frac{r}{1+r} \frac{1}{1+r} T_e.
\]

Unlike a simple repeated game, folk theorem arguments do not directly apply because the analogue of the stage game payoffs also depends on \( r \). To separate out this effect, we will take limits as \( r \) gets small holding

\[
\frac{1+r}{(1+\rho)^2} = k < 1,
\]

constant. This has the effect of keeping fixed \( T_e \).

It is easy to see that in this limit, the left hand side of the inequality converges to a positive constant. The last term on the right hand side converges to zero, and so all the remains is to determine the effect on the first term on the right hand side. But the term in square brackets is positive, and given the way we are taking limits

\[
\lim_{r \to 0} \frac{r}{1+r \rho (2+\rho)} = \lim_{r \to 0} \frac{r}{1+r (1+\rho)^2 - 1} \quad = \quad \lim_{r \to 0} \frac{k}{1+r - k} = 0.
\]

Hence, for all \( N \) there exists an \( r^* \) such that if \( r \geq r^* \) cooperation can be sustained.