Working Paper No. 414

Foreign Exchange Forward Pricing under Hedging Constraints: A Case Study of China’s Foreign Exchange Forward Markets

by

Yi David Wang

March 2010

Stanford University

579 Serra Mall @ Galvez, Landau Economics Building, Room 153

Stanford, CA 94305-6015
Foreign Exchange Forward Pricing under Hedging Constraints: A Case Study of China’s Foreign Exchange Forward Markets

Yi David Wang

March 2010

Abstract

This paper proposes a theoretical framework to answer the question of how foreign exchange forward contracts should be priced when traders face hedging constraints. In particular, when hedging or covering of a foreign exchange forward contract is hindered, either by artificially imposed capital controls or temporary disturbances in the markets, the price of a foreign exchange forward deviates from the covered interest rate parity and instead equals to a linear combination of two factors: the CIP-implied forward price and the expectation of future spot rate, with the weight determined by the level of hedging constraint. Using data of dollar-yuan forwards, there is empirical evidence in support of this hypothesis.

Key words: China, foreign exchange, covered interest rate parity, deliverable forward, non-deliverable forwards

JEL codes: F31, F42

1 Department of Economics, Stanford University, 579 Serra Mall, Stanford, CA, 94305, USA. E-mail: dyiwang@stanford.edu. I am grateful for the guidance and advice of Professor John Taylor and Professor Ronald McKinnon on this paper. All remaining errors are solely my own.
1. Introduction

China’s young onshore interbank foreign exchange forward market has received surprisingly little academic attention since its establishment in October 2005. Although transaction volume data of this wholesale market has been elusive up to this point, there are reasons to believe that a lot of money is at stake. For example, according to an estimate in 2004, even the offshore retail foreign exchange market between the RMB and U.S. dollar had a typical daily volume of about $150 to 200 million while exhibiting an upward trend toward $600 million (Fung et al. 2004). In addition, it has also been documented that forward prices in this interbank market violate the covered interest rate parity (CIP) (Wang 2010 and McKinnon et al. 2010). Furthermore, there also appears to be significant price differences between the onshore-interbank-deliverable forwards and the offshore-retail-non-deliverable forwards. Both the violation of CIP in the interbank market and the price discrepancy between onshore and offshore forwards generate potential arbitrage opportunities to market participants. Given the amount of money at stake, any potential arbitrage opportunities in China’s foreign exchange forward markets deserve careful investigation by both academic researchers in the field of economics and market participants.

Foreign exchange forward pricing (in a frictionless world) is a problem solved long ago. Indeed, covered interest rate parity states that the forward price between two currencies should equal to the spot rate times the interest rate differential of the two currencies (i.e. \( F = S \frac{1 + i_A}{1 + i_B} \) where \( F \) and \( S \) are the forward and spot exchange rate quoted in the units of currency A per 1 unit of currency B). The empirical robustness of CIP
hinges on the proper functioning of three markets: the spot foreign exchange market and
the money markets of the two currencies. As the CIP formula suggests, a trader can
replicate a forward position by borrowing in one currency, convert the borrowing
proceeds into the other currency in the spot market, and lend in the other currency.
Furthermore, the fact that CIP is empirically robust in markets such as the dollar-sterling
and dollar-euro market (with minor deviations attributable to transaction costs) also
diminishes the incentive for academic researchers to develop a forward pricing model
when there are significant frictions in one (or some) of the three markets mentioned
above. After all, there is no point in developing a model when there does not appear to
be a need for it.

However, in China’s case, its measures of capital controls translate into
interventions in the spot market. In particular, China’s State Administration of Foreign
Exchange (SAFE) has the legal authority to review all foreign exchange transactions
taking place in the interbank market. Consequently, no interbank spot transactions can
legally take place without the approval of SAFE. For example, assume that Citi and
HSBC have agreed to conduct a spot conversion between dollar and RMB. In order for
this trade to take place, both parties need to submit proof that such transaction is a current
account transaction (i.e., linked to trading needs). If the transaction is a capital account
transaction, then the parties have to show that this particular capital account transaction is
in line with capital control policies. Unfortunately, attempting to arbitrage the difference
between actual forward rate and CIP-implied forward rate is not an acceptable
justification for the need to carry out a spot trade.
Of course, SAFE’s disapproval does not necessarily mean that the participants would stop trying to arbitrage the CIP violations. After all, other than denying all or part of a transaction, there does not appear to be additional punishments other than perhaps verbal warnings. There has never been a dismemberment of a market participant on the basis of unable to provide proper documentation for SAFE reviewing to my knowledge. Hence, it should be no surprise that many market participants are indeed trying to arbitrage the CIP violation whenever possible, perhaps by masking such arbitrage transactions as current account transactions. Nonetheless, SAFE’s monitoring effort introduces a constraint on the hedging of a foreign exchange forward position. For example, a trader with a position to buy dollars forward might find part of his forward position uncovered because his spot transaction to sell dollars in exchange for RMB has been denied by SAFE.

Facing such hedging constraints, there should be no surprise that actual forward price deviates from the CIP-implied forward price. Indeed, historical violations of CIP in the interbank forward market have been attributed to SAFE intervention in the spot market, especially after SAFE announced to increase its monitoring effort on May 18, 2007 (Wang 2010). Yet, in light of the hedging constraint, a more important and arguably more interesting question is how should foreign exchange forward be priced when traders face government-imposed hedging constraints? This paper provides an answer to this question and uses Chinese forward data to verify the theoretical hypothesis empirically. The main finding is that when hedging constraints are present, interbank forward rate is a weighted average of two prices: the CIP-implied forward rate and the expectation of future spot rate, with the weight reflecting the level of hedging constraints.
Empirically, China’s foreign exchange market data is consistent with this hypothesis. A second theoretical result is that retail forward rates between yuan and dollar should equal to the expectation of future spot rate, which also has some empirical support.

The intuition behind the main finding is straightforward. Imagine two risk-neutral agents entering into a foreign exchange forward contract but neither party is allowed to hedge his position. The only forward rate at which they can strike a deal should be their expectation of the spot rate at maturity.\(^2\) Hence, the CIP-implied forward rate and the expectation of spot rate at maturity should serve as boundaries for the actual forward rate. When there is no hedging constraint, forward rate equals the CIP-implied forward rate. When there is full hedging constraint, forward rate equals the expectation of spot rate at maturity.\(^3\) When there is partial hedging constraint, the intuitive hypothesis is that the forward rate should be somewhere in between \(i.e., F_D = (1-\alpha)F_{CIP} + \alpha E_r[S_{t+1}]\) where \(\alpha\) is the measure of hedging constraint. The rest of this paper attempts to justify this seemingly intuitive hypothesis both theoretically and empirically.

The remaining of this paper is organized in the following fashion. Section 2 provides background information on the relevant markets. Hopefully the descriptions will render certain assumptions in the model more justifiable. Section 3 sets up the model and discusses its theoretical and empirical implications. Section 4 describes the data. Section 5 presents the empirical results. Finally, section 6 concludes and discusses the applicability of this model in other forward markets. Figures are contained in the appendix.

### 2. Background Information

\(^2\) Assuming that they have the same expectation.

\(^3\) Assuming no risk premium
2.1 Retail Dollar-Yuan Forward Markets

The retail dollar-yuan forward markets are offshore\(^4\) markets and the forward contracts traded in the retail markets are non-deliverable forwards. A non-deliverable forward is conceptually similar to an outright forward contract. A notional principal amount, the forward rate, and maturity date are all specified in the contract. On maturity, the two parties do not exchange the currencies. Instead, only a net settlement will be made to reflect the difference between the agreed forward rate and the actual spot rate on maturity. In the case of dollar-yuan retail forwards, the difference is cash-settled in dollars.

There are currently two highly active retail dollar-yuan forward markets: Hong Kong and Singapore. The Singapore retail market dates back to December 1998 while the Hong Kong retail market did not pick up until October 2005.\(^5\) This paper focuses on the Hong Kong market. In October 2005, Hong Kong launched retail dollar-yuan non-deliverable forward contracts. The contracts are offered for a minimum of $10,000. The relative small subscription size of these contracts caters to the hedging needs of small and medium-sized enterprises with RMB exposures in addition to large enterprises.

Different from the retail non-deliverable forward contracts of Singapore, forward maturity and price for the contracts offered in Hong Kong are standardized rather than individually negotiated between offering banks and investors. At the time of introduction

---

\(^4\) Outside mainland China.

\(^5\) There were some OTC trading of forward contracts in Hong Kong prior to October 2005, but the volume was small and estimated to be 5% of all non-deliverable RMB forward contracts (Fung et al. 2004)
of such retail forward contracts, Hong Kong Monetary Authority had designated sixteen banks to offer this service.\textsuperscript{6}

For example, Nanyang Commercial Bank is one of the 16 banks designated to be a market maker in this retail forward market. It offers this product to its customers free of charge. Yet, the customer is required to post certain amount of collateral. Customers can choose to use different types of deposit as the collateral, including fixed deposit, foreign currency savings, Hong Kong dollar savings, current deposits or RMB savings. The minimum contract size is $10,000 and there is no upper limit on the number of contracts a customer can enter.

First, the forward rates of various maturities are made available to the customers. Customers then decide to buy or sell RMB forward base on their hedging needs. At maturity, settlement amount is equal to contract notional amount x [1-forward rate/settlement rate]. The settlement rate is defined as the official closing exchange rate for RMB against the U.S. dollar as announced by the People’s Bank of China on valuation date. If the settlement amount is greater than zero, then the seller of dollar shall pay the settlement amount to the buyer. If the settlement amount is less than zero, then the buyer of dollar shall pay the settlement amount to the seller. Foreign currency savings account (usually a U.S. dollar savings account) is used for settlement purpose. If customers do not have foreign currency savings account, his/her Hong Kong Dollar savings account, current account or RMB saving account will be considered as settlement account after currency conversion.

\textsuperscript{6} The sixteen banks are: Bank of China, Bank of Communications, Bank of East Asia, Chiyu Bank, Citibank (Hong Kong), DBS Bank, Fubon Bank, Hang Seng Bank, HSBC, Industrial & Commercial Bank of China, Liu Chong Hing Bank, Nanyang Commercial Bank, Royal Bank of Scotland, Shanghai Commercial Bank, Standard Chartered Bank, and Wing Lung Bank. Information regarding the Hong Kong retail non-deliverable forward market is reported by a China Daily article dated September 27, 2005.
2.2 Wholesale Dollar-Yuan Spot and Forward Markets

The official interbank foreign exchange market in China is called China Foreign Exchange Trade System (CFETS). It is the only legal market for interbank foreign exchange activities in China, and participation is restricted to members only. It was initially founded in February 1994, which marked the unification of the highly fragmented inter-bank foreign exchange markets in China. CFETS headquarter is in Shanghai, with a backup headquarter in Beijing. In addition, it also has 18 sub-centers.\(^7\) It is a sub-department of People’s Bank of China (PBoC), and is also regulated by the State Administration of Foreign Exchange (SAFE).

All foreign exchange transactions are required to occur in CFETS during its market-hours,\(^8\) hence no after hour trading is allowed. Currently, the products being traded on CFETS include spot trading, deliverable forward contracts between RMB and USD, and currency swaps between RMB and other foreign currencies. For the spot market, the major currencies involved are RMB, US dollar, Sterling, Hong Kong Dollar, Yen, and Euro. Forward contracts between RMB and US dollar were first introduced in October 2005, less than 3 months after China ended its decade long pegging regime. There are currently 279 members in the spot market and 69 members in the forward market (CFETS 2009b).\(^9\) Out of the existing members, there are 16 that serve as primary market makers in both spot and forward markets.\(^10\) Out of these 16 members, 6 are also market makers in the retail forward market.\(^11\)

---

\(^7\) The 18 cities are Guangzhou, Shenzhen, Tianjin, Jinan, Dalian, Nanjing, Xiamen, Qingdao, Wuhan, Chongqing, Chengdu, Zhuhai, Shantou, Fuzhou, Ningbo, Xian, Shenyang, and Haiko
\(^8\) 9:30-15:30 Monday through Friday with the exception of holidays
\(^9\) As of 11/24/2009
\(^10\) The 16 primary market makers are: Australia and New Zealand Banking Group; Bank of China; Bank of Montreal; Bank of Tokyo-Mitsubishi UFJ; Banque Indosuez; BNP Paribas; China CITIC Bank; China
Spot trade in CFETS can occur in two ways. The first one is that traders report an order (price and volume) into an electronic system, then the computer matches the highest bid and the lowest ask. Traders can also log into this system without reporting an order to obtain quotes and follow the market. The second way is by individual negotiation. Members can directly trade with a market maker or with any other member, all transactions have to be approved by SAFE prior to its execution and the final transaction will be reported for record keeping. For forward contracts and currency swaps, only the second method is allowed.

Chinese authorities intervene heavily in the foreign exchange spot market. Take the dollar-yuan spot market for example, the PBoC sets the opening quote based on the previous day’s closing price and allows a narrow range of daily fluctuation. When supply and demand forces require the market clearing price to go beyond the fluctuation range, the PBoC steps in to buy (or sell) dollars to maintain the exchange rate within the band. At the end of the trading day, the PBoC also announces the official closing rate. Another way of intervention comes from SAFE monitoring. In particular, all interbank foreign exchange transactions require SAFE approval prior to its completion. A transaction will be blocked if SAFE does not approve it.

Forward contracts in this market are deliverable contracts with standardized maturities. Yet, a forward contract in the wholesale market can be settled in two ways: gross settlement and net settlement. Under gross settlement, full notional amounts of the

---

Construction Bank; Citi Bank; Deutsche Bank; HSBC; Industrial and Commercial Bank of China; ING Bank; Standard Chartered Bank; UBS; Royal Bank of Scotland.


Forward contracts of 1 week, 1 month, 3 month, 6 month, 9 month, and 12 month are traded. Forwards with longer maturities do exist, but are less liquid.
yuan and dollar are exchanged on delivery date (i.e. contract maturity date). Under net settlement, counter parties only settle the difference between the forward rate and settlement rate. The net settlement can be settled in either dollar or RMB, which is agreed upon in advance. The settlement rate is defined as the closing rate two days prior to maturity date. Settlement method is agreed upon in advance by the two counter parties when they enter into the contract.

Counterparties in the forward contracts can require collaterals from each other. The collateral amount, delivery date, and returning date are determined by the two counterparties on a case-by-case basis. The CFETS can provide the safekeeping service and hold the collaterals if the two counter parties desire such arrangement. Yet, CFETS does not require the posting of collaterals. Collateral can be denominated in any currency as long as the two parties agree. Although exact figures are not available, one would guess that for dollar-yuan forward contracts, the collateral currencies (if any) are dollar and/or RMB.

The primary purpose of these interbank forward contracts is to allow banks to provide better hedging instruments to their customers (CMPR 2007Q2)\textsuperscript{13}. Hence, it appears that the intended objective of the interbank wholesale forward market is to allow banks to cover any net forward positions they have accumulated in the retail market with their customers.

2.3 China’s Interbank Money Market

The official interbank money market for RMB borrowing and lending is the National Interbank Funding Center (NIFC). The NIFC was officially established in

\textsuperscript{13} “CMPR” is short for China Monetary Policy Report, see reference for further details
January 1996, under a mandate by the PBoC that required all inter-bank borrowing and lending activities be carried out via the NIFC. Furthermore, on January 3rd 1996, NIFC and CFETS became de facto one market in the sense that both locate in the same physical location and use the same operating system. Finally, the de facto combination of NIFC and CFETS is officially recognized by a PBoC mandate on January 27, 1997. Hence, both CFETS and NIFC are regulated by the PBoC and treated as one entity.

For unsecured lending and borrowing among financial institutions, the relevant interest rate is the Shanghai Interbank Offered Rate (Shibor). Conceptually, Shibor is equivalent to Libor with the exception that the market is physically located in Shanghai instead of London. In particular, it is a simple, no-guarantee, wholesale interest rate calculated by arithmetically averaging all the interbank RMB lending rates offered by the price quotation group of banks with a high credit rating. There are currently 16 banks in the quoting group.14 Although all Shibor-reporting banks are participants in the CFETS, only 6 of them are also primary market makers in CFETS. In addition, only 5 are market makers in the retail forward market. The intersection of all three lists consists of 4 banks: Bank of China, HSBC, ICBC, and Standard Chartered.

Shibor is calculated by removing the top 2 and bottom 2 rates and then averaging the remaining 12 quotes. Currently, the Shibor survey banks are required to provide rates on the following eight maturities: overnight, 1-week, 2-week, 1-month, 3-month, 6-month, 9-month and 1-year. In addition to the required rates, reporting banks also have the option to report any of the following eight maturities: 3-week, 2-month, 4-month, 5-

---

14 The 16 reporting banks for 2009 are: Agricultural Bank of China; Bank of Beijing; Bank of China; Bank of Communications; Bank of Shanghai; China CITIC Bank; China Construction Bank; China Everbright Bank; Shanghai Pudong Development Bank; China Merchants Bank; HSBC; Huxia Bank; Industrial and Commercial Bank of China; Industrial Bank Co., Ltd.; Postal Savings Bank of China; Standard Chartered Bank.
month, 7-month, 8-month, 10-month, and 11-month. The rates are quoted in percentage as annual rates using 360 days per year and retain four digits after the decimal.

3. The Model

3.1 Basic Setup

There are two currencies: U.S. dollar and Chinese yuan (RMB).

There are two types of foreign exchange forward markets: the interbank wholesale market and the retail market.

There are three types of players: retail customers, retail banks, and wholesale banks.

There are three other interbank markets that the model treats as exogenous: the foreign exchange spot market, the dollar money market, and the yuan money market. In particular, the three types of players mentioned above take the market clearing prices of these three markets as exogenously given when making decisions.

The retail customers have no access to the interbank market, and hence can only participate in the retail market. They do not have access to the interbank money markets of either currency.

The retail banks are the market makers in the retail market. Forward contracts in the retail market are made between retail customers and retail banks. In particular, retail banks cannot enter into forward contracts with each other (or with other banks) in the retail market. In addition, retail customers cannot enter into forward contracts with each other. Given that the retail banks are banks, they are also participants of the interbank wholesale market and the interbank money markets of the two currencies. Any net forward position they have accumulated with retail customers in the retail market can in
turn be covered in the wholesale market or internally. For example, if a retail bank has build up a net forward position to deliver yuan and receive dollar on a certain date, then it can cover this position by entering into a forward contract to deliver dollar and buy yuan in the wholesale market. Alternatively, it can hedge internally by borrowing dollar in the dollar money market now, convert the dollar into yuan in the spot exchange market, and lend the yuan in the Chinese money market up to the deliver date of its retail forward contract. The latter method is referred to as internally covering a forward position in this paper.

Given that retail customers do not have access to the interbank money market, it is difficult for them to replicate a forward position. In other words, the retail market is the only way for the retail customers to hedge away their foreign exchange risks.

Finally, the wholesale banks include two groups: market makers and non-market-making participants. Both of these groups can include retail banks.

Assumption 1: Let $\alpha \in [0,1]$ be the measure of internal-covering constraint associated with a forward contract faced by a bank.

In particular, for every dollar transacted in a forward contract, assume that the trader can only internally cover $(1-\alpha)$ dollar. Hence, $\alpha = 0$ means the ability to fully cover a forward contract (i.e., no covering difficulty). Similarly, $\alpha = 1$ means that a forward contract cannot be hedged internally at all. As discussed in Introduction, the covering constraint in the yuan-dollar market originates primarily from SAFE’s monitoring of the spot market.

Assumption 2: All wholesale forward market participants are risk neutral.
**Assumption 3:** Retail banks are risk-neutral. When expected payoffs between two trades are identical, retail banks resolve their indifference in favor of the trade that has a lower payoff variance.

**Assumption 4:** Retail banks and other participants in the wholesale forward market take the spot exchange rate \( S_t \), interbank interest rates for the two currencies \( (i_{RMB}, i_s) \), and the current expectation of future spot rate conditional on all currently available information \( E_t[S_{t+1}] \) as predetermined state variables when deciding wholesale forward rate \( F_D \) and retail forward rate \( F_{ND} \).

Given that the spot exchange rate is controlled by the PBoC, and PBoC probably does not cater to the need of any particular bank, it is reasonable to assume that banks take \( S_t \) as exogenous. Despite the fact that forward rate can definitely lead to capital movement in search of arbitrage opportunities, which might influence interbank interest rates depending on the size of the capital movement, as long as forward traders take the interest rates as given when determining forward rates, the predetermination of \( i_{RMB}, i_s \) is a defendable assumption. Indeed, when a forward trader from Citi is about to provide a quote for a forward position, it is more likely that he glances at the interbank interest rates on his Bloomberg screen before providing the quote. It is unlikely that any forward trader believes that his decision to enter into a forward position would have noticeable impact on the interbank interest rates.

The assumption of \( E_t[S_{t+1}] \) being predetermined appears the least defendable. In fact, it is very likely that traders might even use forward prices to update their expectation. Hence, the formation of \( E_t[S_{t+1}] \) probably happens concurrently with the formation of
forward prices. This is a weakness in my model. A more defendable model should also incorporate the formation of market expectation of future spot rate, which I plan to attempt in the future.

Retail banks would announce the retail forward price ($F_{ND}$). Taking $F_{ND}$ as given, retail customers enter into forward contracts with retail banks, and retail banks realize whether they are net dollar buyers or net dollar sellers in the retail forward market. Each retail bank decides whether it would like to cover its net position. If yes, it decides whether it will cover via the wholesale market or internally based on wholesale forward rate.

**Assumption 5:** When announcing $F_{ND}$, retail banks are completely agnostic about whether they will become net buyers or net sellers in the retail market. In particular, they do not know the probability distribution function of their net positions in the retail market and act under the belief that their choice of $F_{ND}$ cannot influence the probability distribution function.

As described in the previous section, retail banks offer forward contracts to their clients (i.e., account holders). Once $F_{ND}$ is announced, the retail bank needs to be prepared to take on either side of the contract. Given that the customer base are made up by small to mid-size enterprises with dollar-RMB exposures, which are most likely firms involved in import and export, the positions these customers will take are beyond control of their retail bank. In particular, a price change in $F_{ND}$ will not change the forward position a Chinese firm that exports shoes to the U.S. would want to take, for this firm is not in the business of making profits in the retail forward market, but is only interested in the forward contract as a mean to hedge its dollar-RMB exchange rate risk. Hence, it
appears reasonable to assume that the retail banks take the outcome of its net forward position as exogenous.

**Theoretical Results**

**Proposition 1:** Under Assumptions 1, 2, and 4, wholesale forward pricing is described by Equation (1)

\[
F_D = (1 - \alpha) F_{CIP} + \alpha E_t[S_{t+1}]
\]

Where \( F_{CIP} = S_t \frac{1 + i_{RMB}}{1 + i_S} \) with \( S_t \) being the spot exchange rate in the interbank market, \( i_{RMB} \) being the interest rate of Chinese yuan in the interbank RMB money market, and \( i_S \) being the interest rate of U.S. dollar in the interbank dollar money market.

**Proof:**

Consider any pair of dollar buyer and dollar seller in the wholesale forward market. By agreeing on \( F_D \), the dollar seller will gain \( F_D - F_{CIP} \) for the fraction of the proceeds he can internally hedge. In addition, he will expect to gain \( 1 \) on the fraction he cannot hedge internally. Given that he is assumed to be risk neutral (Assumption 2), he will enter into such a contract if and only if

\[
(1 - \alpha)(F_D - F_{CIP}) + \alpha(F_D - E_t[S_{t+1}]) \geq 0,
\]

or equivalently

\[
F_D \geq (1 - \alpha) F_{CIP} + \alpha E_t[S_{t+1}]
\]

By symmetry, the dollar buyer in this transaction will enter into such a contract if and only if

\[(1 - \alpha)(F_{CIP} - F_D) + \alpha(E_t[S_{t+1}] - F_D) \geq 0 \]

or equivalently

\[
F_D \leq (1 - \alpha) F_{CIP} + \alpha E_t[S_{t+1}]
\]
Combining (1.1) and (1.2), the only forward rate at which both parties are willing to transact at is

$$F_D = (1 - \alpha)F_{CIP} + \alpha E_t[S_{t+1}]$$

QED.

Proposition 1 says that when a forward position cannot be fully covered internally, the forward price deviates from the no-arbitrage price ($F_{CIP}$) and shift towards the expectation of future spot price ($E_t[S_{t+1}]$). If a forward can be fully covered internally, then forward price equals $F_{CIP}$. On the other hand, if a forward cannot be covered internally at all, forward price is determined solely by the expectations of future spot price. Finally, when a forward can only be partially covered internally, its pricing depends on the weighted average of $F_{CIP}$ and expectation of future spot price.

Now that we have resolved the question of how wholesale forwards are priced, we turn next to the question of locating an observable measure for $E_t[S_{t+1}]$ by examining the retail forward market. First, Proposition 2 below lists conditions under which a retail bank would want to cover its net forward position in the retail market via the wholesale forward market.

**Proposition 2:** Under the same assumptions 1-4, a retail bank that is a net dollar buyer in the retail market would cover its position in the wholesale market if and only if $F_{CIP} \geq E_t[S_{t+1}]$. Similarly, a retail bank that is a net dollar seller in the retail market would cover its position in the wholesale market if and only if $F_{CIP} \leq E_t[S_{t+1}]$.

**Proof:**

If a retail bank that has accumulated a net dollar-buying position in the retail forward market and decides to cover via the wholesale forward market, it faces a payoff
of $F_D - F_{ND}$. By Proposition 1, this payoff is equal to $(1-\alpha)F_{CIP} + \alpha E_i[S_{t+1}] - F_{ND}$. If it does not cover its retail position, then its expected payoff is $E_i[S_{t+1}] - F_{ND}$. Given that the retail bank is risk neutral, it only cares about the expected payoffs when deciding whether or not to cover (Assumption 2). Consequently, it covers its retail position if and only if

$$(1-\alpha)F_{CIP} + \alpha E_i[S_{t+1}] - F_{ND} \geq E_i[S_{t+1}] - F_{ND}$$

$\iff (1-\alpha)F_{CIP} \geq (1-\alpha)E_i[S_{t+1}]$,

$\iff F_{CIP} \geq E_i[S_{t+1}]$.

The second part of Proposition 2 regarding the net dollar seller in the retail forward market can be proved by symmetry.

QED

Now let us turn our attention to the determination $F_{ND}$ and its relationship to $E_i[S_{t+1}]$, which is summarized in Proposition 3.

**Proposition 3:** Under Assumptions 1-5, $F_{ND} = E_i[S_{t+1}]$.

*Proof:*

When a retail bank announces $F_{ND}$, $F_{CIP}$ and $E_i[S_{t+1}]$ are common knowledge. So there are three scenarios to analyze, $F_{CIP} < E_i[S_{t+1}]$, $F_{CIP} > E_i[S_{t+1}]$, and $F_{CIP} = E_i[S_{t+1}]$.

Assume that $F_{CIP} < E_i[S_{t+1}]$, then by Proposition 2, a retail bank with a net position to sell dollar in the retail market would cover its position while a retail bank with a net position to buy dollar in the retail market would not cover its position. By Assumption 5, retail banks do not know the probability distribution of whether they will become net buyers or net sellers of dollar in the retail forward market, hence they do not know the

---

15 If the retail bank decides to cover its position internally, it would also receive this payoff.
unconditional expected payoff when they announce $F_{ND}$. Consequently, they will focus on the expected payoff conditional on their net positions when determining $F_{ND}$.

Under the current scenario, conditional on being a net seller of dollar in the retail market, a retail bank’s payoff is $F_{ND} - (1 - \alpha)F_{CIP} - \alpha E_t[S_{t+1}]$ because it will cover its position. Conditional on being a net buyer of dollar in the retail market, the retail banks expected payoff is $E_t[S_{t+1}] - F_{ND}$ because it will not cover its position. By Assumption 3, the retail bank is willing to trade at $F_{ND}$ as long as the conditional expected payoffs are greater than or equal to zero.

Conditional on being a net buyer of dollar in the retail market, the retail bank would announce $F_{ND}$ such that $E_t[S_{t+1}] - F_{ND} \geq 0$. Conditional on being a net seller of dollar in the retail market, the retail bank’s objective is to maximize $F_{ND} - (1 - \alpha)F_{CIP} - \alpha E_t[S_{t+1}]$. Hence, the retail banks optimization problem prior to it finds out whether it is a net buyer or net seller of dollar in the retail market can be summarized as maximize $F_{ND} - (1 - \alpha)F_{CIP} - \alpha E_t[S_{t+1}]$ with respect to $F_{ND}$ such that $E_t[S_{t+1}] - F_{ND} \geq 0$. The solution to this maximization problem is $F_{ND} = E_t[S_{t+1}]$.

The case of $F_{CIP} > E_t[S_{t+1}]$ can be analyzed in a similar fashion with a few minor changes. Assume that $F_{CIP} > E_t[S_{t+1}]$, then by Proposition 2, a retail bank with a net position to sell dollar in the retail market would not cover its position while a retail bank with a net position to buy dollar in the retail market would cover its position. Once again by Assumption 5, retail banks focus on the expected payoff conditional on their net positions when determining $F_{ND}$.
Conditional on being a net seller of dollar in the retail market, a retail bank’s expected payoff is \( F_{ND} - E_t[S_{t+1}] \). Conditional on being a net buyer of dollar in the retail market, the retail bank’s payoff is \( (1 - \alpha) F_{CIP} + \alpha E_t[S_{t+1}] - F_{ND} \). Consequently, the retail banks optimization problem prior to it finding out whether it is a net buyer or net seller of dollar in the retail market can be summarized as \( \max_{F_{ND}} (1 - \alpha) F_{CIP} + \alpha E_t[S_{t+1}] - F_{ND} \) such that \( F_{ND} - E_t[S_{t+1}] \geq 0 \). The solution to this maximization problem is also \( F_{ND} = E_t[S_{t+1}] \).

Finally, when \( F_{CIP} = E_t[S_{t+1}] \), a retail bank with a net position in the retail market would cover regardless of the direction of its net position. Conditional on being a net buyer of dollar in the retail market, a retail bank is willing to trade if and only if \( (1 - \alpha) F_{CIP} + \alpha E_t[S_{t+1}] - F_{ND} \geq 0 \). Similarly, conditional on being a net seller of dollar in the retail market, a retail bank is willing to trade if and only if \( F_{ND} - (1 - \alpha) F_{CIP} - \alpha E_t[S_{t+1}] \geq 0 \). The only value of \( F_{ND} \) that concurrently satisfies both inequalities is \( F_{ND} = E_t[S_{t+1}] \).

QED

3.2 Empirical Implications

An immediate consequence of Proposition 1 and Proposition 3 is

\( F_D = (1 - \alpha) F_{CIP} + \alpha F_{ND} \),

which says that the forward price in the interbank wholesale market should be a weighted average of the no-arbitrage price \( F_{CIP} \) and the forward price in the retail market \( F_{ND} \). The weight is determined by the difficulty to cover internally a forward position faced by wholesale market participants. This naturally leads to a regression.
Regression (1) \[ F_{D,t} = \beta_1 F_{CIP,t} + \beta_2 F_{ND,t} + \epsilon_t \]

Hence, conditional on the assumption that covering difficulty does not change during a time interval, \( \alpha \) can be estimated by \( \hat{\beta}_2 \) from Regression (1). In addition, \( \hat{\beta}_1 + \beta_2 \approx 1 \). Furthermore, given that SAFE announced on May 18, 2007 to increase its monitoring effort of the spot foreign exchange market, which should have increased the covering difficulty faced by members of the interbank wholesale forward market, there should be an increase in \( \hat{\beta}_2 \) from prior to the announcement to after the announcement, which is equivalent to an equal decrease in absolute magnitude in \( \hat{\beta}_1 \).

In addition, according to Proposition 3, \( F_{ND} = E_t[S_{t+1}] \). If Proposition 3 is indeed correct, then the average deviation between retail forward rates and spot rates at maturity should approach zero as the number of observations approaches infinity.

\[
\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} F_{ND,t} - S_{t+1} = 0
\]

4. Data Description

From Regression (1), three sequences of historical data are needed to verify the model empirically. In particular, they are the forward price in the interbank wholesale market \( (F_D) \), the forward price implied by covered interest rate parity \( (F_{CIP}) \), and the forward price in the retail market \( (F_{ND}) \). Given that \( F_{CIP} = S_t \frac{1 + i_{RMB}}{1 + i_s} \), to generate \( F_{CIP} \), spot exchange rate and proxies for the two interest rates are also necessary.

The interbank dollar-yuan forward market started trading on October 19th, 2005, and daily forward prices of various forward maturities are provided by the China Foreign
Exchange Trade System (CFETS).\textsuperscript{16} To generate $F_{CIP}$, the historical spot exchange rates between dollar and yuan are obtained from SAFE. For the two interest rates, Shibor is used as a proxy for $i_{RMB}$ and the dollar Libor as a proxy for $i_s$.\textsuperscript{17} The source of Shibor is China’s National Interbank Funding Center (NIFC). The dollar Libor is obtained from Bloomberg. Finally, both Hong Kong and Singapore have retail forward markets in which small and medium-sized enterprises can enter into dollar-yuan forward contracts with banks that serve as market makers.\textsuperscript{18} The historical dollar-yuan forward prices in these retail markets are also available on Bloomberg. Figures 1-5 illustrates the three historical forward prices ($F_D, F_{CIP}, F_{ND}$), along with historical spot price, from October 19\textsuperscript{th}, 2005 to October 23\textsuperscript{rd}, 2009 for 1-month, 3-month, 6-month, 9-month, and 12-month forwards respectively.\textsuperscript{19}

Equation (1) can also be manipulated to show the relationship between deviations from CIP in the wholesale and retail market.

\[ F_D = (1 - \alpha)F_{CIP} + \alpha F_{ND} \Rightarrow \frac{F_D}{F_{CIP}} = (1 - \alpha) + \alpha \frac{F_{ND}}{F_{CIP}} \]

\[ \Rightarrow \frac{F_D}{F_{CIP}} - 1 = \alpha \left( \frac{F_{ND}}{F_{CIP}} - 1 \right) \]

\textsuperscript{16} For the purpose of this paper, forward maturities of 1 month, 3 month, 6 month, 9 month, and 12 month are used.
\textsuperscript{17} Justifications for the usage of these two proxies are provided in an earlier chapter (Wang 2010).
\textsuperscript{18} The forwards traded in these offshore markets are non-deliverable forwards. The primary purpose of these non-deliverable forwards is to allow businesses to hedge their dollar-yuan exchange rate risks. It also allows individual investors to bet on future spot rates between dollar and yuan (Fung et al 2004).
\textsuperscript{19} Interest rates used in the calculation of $F_{CIP}$ for different forward maturities are interest rates with matching length. For example, the 1-month $F_{CIP}$ is calculated using the 1-month Shibor and 1-month Libor. For dates prior to October 8\textsuperscript{th}, 2006, given that Shibors are not publicly available, People’s Banks of China’s Deposit Rate for Financial Instituions (金融机构人民币存款基准利率) is used in place of Shibors.
Notice that the left hand side of the expression above is the deviation from CIP of the interbank wholesale forward. The expression in the parenthesis on the right hand side is the deviation from CIP of the retail forward. If there is a-priori belief that $\alpha$ has changed on a certain date, then such change should be reflected visually in scatter graphs of the two types of deviations. Figure 6-10 are scatter graphs of CIP deviations exhibited by the two markets using forwards of various maturities. From visual inspection, there appears to be an increase in the correlation between the two types of deviations after the SAFE announcement on May 18th, 2007.

5. Empirical Results

Table 1 shows the results of Regression (1). Due to the SAFE announcement on May 18th, 2007, there is reason to believe that there was an increase in $\alpha$ on that date. Consequently, the sample is divided into two periods: pre-announcement and post-announcement. Table 1 shows the estimation results for Regression (1) by forward maturities.

Regression (1) \[ F_{D,t} = \beta_1 F_{CIP,t} + \beta_2 F_{ND,t} + \epsilon_t \]

Table 1: Regression Results for Regression (1) by Forward Maturities

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta1</td>
<td>0.6646*</td>
<td>0.5703*</td>
<td>0.6704*</td>
<td>0.7395*</td>
<td>0.3200*</td>
<td>0.7731*</td>
<td>0.2567*</td>
<td>0.8151*</td>
<td>0.2547*</td>
<td></td>
</tr>
<tr>
<td>Beta2</td>
<td>0.3969*</td>
<td>0.5986*</td>
<td>0.3303*</td>
<td>0.5822*</td>
<td>0.2913*</td>
<td>0.6796*</td>
<td>0.2264*</td>
<td>0.7440*</td>
<td>0.1846*</td>
<td>0.7463*</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9877</td>
<td>0.9863</td>
<td>0.9811</td>
<td>0.9811</td>
<td>0.9811</td>
<td>0.9811</td>
<td>0.9811</td>
<td>0.9811</td>
<td>0.9811</td>
<td>0.9811</td>
</tr>
</tbody>
</table>

* Significant at 1% level

The results are consistent with the model predictions. First, without imposing the condition of $\hat{\beta}_1 + \beta_2 = 1$, the regression results for the two coefficients do sum up close to 1. Furthermore, there was indeed an increase in $\hat{\beta}_2$ during the post announcement period,
and such increase is robust to forward maturities. Figures 11-15 illustrates the actual and fitted wholesale forward prices (along with 95%-confidence regions) using the estimation results above. The discrepancies between fitted and actual wholesale forward prices are small, and the actual forward prices are always within the confidence region. The discrepancies appear to be increasing with forward maturity.

As a robustness check, there is an alternative estimator for $\alpha$, which originates from the relationship between deviations from CIP in the wholesale and retail market. It has been shown that

$$
(2) \quad \frac{F_{D,i}}{F_{CIP,i}} - 1 = \alpha \left( \frac{F_{ND,i}}{F_{CIP,i}} - 1 \right)
$$

Equation (2) leads to the regression below.

Regression (2)  
$$
\frac{F_{D,i}}{F_{CIP,i}} - 1 = c + \beta_2 \left( \frac{F_{ND,i}}{F_{CIP,i}} - 1 \right) + \epsilon_i
$$

Table 2: Regression Results for Regression (2) by Forward Maturities

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_2$</td>
<td>0.3354*</td>
<td>0.4446*</td>
<td>0.2561*</td>
<td>0.6761*</td>
<td>0.2547*</td>
<td>0.6716*</td>
<td>0.2331*</td>
<td>0.7379*</td>
<td>0.1939*</td>
<td>0.7376*</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.210</td>
<td>0.5376</td>
<td>0.3519</td>
<td>0.7495</td>
<td>0.2906</td>
<td>0.8332</td>
<td>0.2395</td>
<td>0.9365</td>
<td>0.1645</td>
<td>0.9367</td>
</tr>
</tbody>
</table>

Comparing $\hat{\beta}_2$ from Table 1 and Table 2, the two estimates are similar. Results of Regression (2) also reflect an increase in covering difficulty post the SAFE announcement.

Finally, Proposition 3 implies

$$
(3) \quad \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} F_{ND,i,t} - S_{i+1} = 0
$$
Given that the model made no assumptions on the data generating process for the random variable $S_{r=1}$, it is meaningless to discuss whether the sample average differences between retail forward rates and spot rates at maturity are significantly different from zero at this point without imposing more structure. However, table 3 lists the sample average differences by forward maturities along with the sample standard deviations for the differences. Although the sample averages do not equal zero, none of the averages are off the mark by more than a penny.\textsuperscript{20} In addition, zero is within 1/10 to 1/5 standard deviations from the sample averages. Given the closeness of the sample averages to zero, there is reason to believe that retail forward prices indeed equal to the expectation of future spot rates.

Table 3: $\frac{1}{n} \sum_{t=1}^{n} F_{ND,t} - S_{r=1}$ by forward maturities

<table>
<thead>
<tr>
<th></th>
<th>1M Mean</th>
<th>1M Std Dev</th>
<th>3M Mean</th>
<th>3M Std Dev</th>
<th>6M Mean</th>
<th>6M Std Dev</th>
<th>9M Mean</th>
<th>9M Std Dev</th>
<th>12M Mean</th>
<th>12M Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1M</td>
<td>0.0056</td>
<td>0.0026</td>
<td>0.0119</td>
<td>0.0029</td>
<td>0.0216</td>
<td>0.1207</td>
<td>0.00278</td>
<td>0.1791</td>
<td>0.021</td>
<td>0.2333</td>
</tr>
<tr>
<td># Obs</td>
<td>1038</td>
<td>981</td>
<td>916</td>
<td>857</td>
<td>786</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With the empirical evidence illustrated so far, the data appear to be consistent with the model implications.

6. Conclusion

Capital control measures in China, along with any market frictions they generate, are part of life. One particular friction generated by the capital control measures is a hedging constraint for traders to internally hedge their forward positions. Consequently, in order to better understand the pricing of dollar-yuan forwards, a defendable and convincing model of foreign exchange forward pricing under hedging constraints is

\textsuperscript{20} Given that the averages are measured in yuan.
necessary. As China, and other developing countries that are also building their foreign exchange markets while implementing capital controls, gains importance in the world economy, such a pricing theory would become more important and valuable with wider applications, rather than simply functioning as a mental exercise to satisfy the curiosity of an inquisitive mind. This paper takes the initial attempt to provide such a pricing model. The primary result is that under hedging constraints, forward price becomes a linear combination of two factors: the CIP-implied forward price and the expectation of future spot rate. Using Chinese data, there is evidence supporting this hypothesis empirically. A secondary result is that non-deliverable forward rates are measures of the market’s expectations of future spot rates.

In addition, the pricing of foreign exchange forward contracts under hedging constraints is applicable not only to countries that impose artificial hedging constraints via capital control measures such as China, but also applicable to well developed foreign exchange markets among countries with little artificially imposed capital control. For instance, violations of CIP in the dollar-euro market around late 2008 have been documented (Reference?). It is possible that these deviations are also results of hedging constraints. Although the constraints did not originate from government intervention in the spot market as it did in China, it might have originated from temporary disturbances in the dollar money markets during the financial crisis. The model presented in this paper might serve as a theoretical starting point for other scholars who wish to study deviations from CIP in forward markets all around the world.

Finally, this paper does not address the important question of how the market’s expectation for future spot rate is formed by assuming $E_t[S_{t+1}]$ as exogenously given. A
more complete and enlightening model should touch on the formation of \( E_j[S_{\tau_1}] \). With
the result of \( F_{\text{no}} = E_j[S_{\tau_1}] \), I will attempt to answer this question in the future.
Figures

Figure 1:  

Sources: CFETS (2009a), NIFC (2009), British Bankers’ Association (2009) and SAFE (2009).

Figure 2:  

Sources: CFETS (2009a), NIFC (2009), British Bankers’ Association (2009) and SAFE (2009).
**Figure 3:**

**Sources:** CFETS (2009a), NIFC (2009), British Bankers’ Association (2009), HK NDF Market (2009) and SAFE (2009).

**Figure 4:**

**Sources:** CFETS (2009a), NIFC (2009), British Bankers’ Association (2009), HK NDF Market (2009) and SAFE (2009).
**Figure 5:**

![Graph of 12 Month Forwards]

**Sources:** CFETS (2009a), NIFC (2009), British Bankers’ Association (2009), HK NDF Market (2009) and SAFE (2009).

**Figure 6:**

![Graph of 1 Month Scatter]

**Sources:** Sources: CFETS (2009a), NIFC (2009), British Bankers’ Association (2009), HK NDF Market (2009) and SAFE (2009).
Figure 7:


Figure 8:

Figure 9:


Figure 10:

Figure 11:

Actual vs. Fitted Wholesale Forward Prices (1 Month)

Figure 12:

Actual vs. Fitted Wholesale Forward Prices (3 Month)
Figure 13:

Actual vs. Fitted Wholesale Forward Prices (6 Month)

Figure 14:

Actual vs. Fitted Wholesale Forward Prices (9 Month)
Figure 15:

Actual vs. Fitted Wholesale Forward Prices (12 Month)
References


