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Should Chinese Citizens be allowed to Invest in China’s Corporate Bond Market?

by

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Abstract

Currently, there is almost no participation from private individuals in China’s corporate bond market and corporate bonds are primarily held by China’s large commercial banks. This paper presents a theoretical proof that such an arrangement is suboptimal from the individual’s perspective. To improve investor welfare, authorities should implement policies to facilitate individual investment in corporate bond. Policy objectives should include (but are not limited to) lowering entry barriers to the bond market faced by potential issuers, establishing proper information/risk disclosure requirements, increasing secondary market liquidity, and increasing the enforceability of bankruptcy laws. Such policies would also mitigate problems associated with China’s “shadow banking system” and are also consistent with China’s long term policy goal of internationalizing the RMB.

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Keywords: China’s corporate bond markets; China’s shadow banking system; Internationalize the RMB

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1. Introduction

China’s corporate bond market has experienced tremendous growth since 2000. Although total issuance size is still small compared to developed markets such as those of the U.S. and Europe (Zeng 2009), China’s corporate bond market has the potential to evolve into a sizable market in the future. Benefiting from the efforts of Chinese authorities to internationalize the RMB (Zhang 2009, Zhou 2009), China’s corporate bond market can eventually function as a source of capital not only to Chinese companies but also to foreign companies as well. With proper development, Chinese corporations can gain access to capital not only from Chinese investors but also from debt investors from around the world who prefer the relative safety of bonds over equity.

Despite a promising future, the truth is that China’s corporate bond market is still underdeveloped in many ways. One serious side effect of this underdevelopment is the existence of a shadow banking system that is relatively opaque and unregulated. This problem, if left unaddressed, could become a drag in China’s long term growth. This drag is not purely a theoretical possibility that may or may not take place in the distant future. Indeed, events in Wenzhou in the second half of 2011 highlight the urgency for authorities to address the shadow banking system. Further developing China’s corporate bond market is a policy solution to mitigate China’s shadow banking problem and is also consistent with China’s long term policy aim of internationalizing the RMB.

A more advanced corporate bond market offers many advantages to China. First, it allows for another channel by which corporations are able to access the capital markets
(Chen et al 2011). This provides competition and flexibility that reduce the issuing costs for both SME and SOEs. Second, a corporate bond market provides competition to lending firms by fostering legitimate sources of debt financing, reducing the stigma associated with borrowing at the corporate level (Zhou 2006). Third, it provides pricing signals to firms enabling them to decide on whether bank finance, corporate bond issuance, or other capital raising alternatives is more efficient (Diamond 1991). Fourth, the public bond prices will provide signals as the riskiness of corporate issuers (Rajan 1992, Sengupta 1998). These signals are important to equity holders, investors, and regulators (Zhou 2006). Fifth, a well developed corporate bond market would stimulate the growth of private investments in China. Private equity (PE) firms in the west as a matter of course use public debt in conjunction with private debt issuance to incentivize management to enhance the value of their firms. Moreover, a public bond market allows private capital to allocate returns more efficiently between equity holders and bond holders. All equity financing for cash-flow producing firms might not be in the best interest of investors who want more growth oriented investments and would prefer that the firm borrows money from them to enhance returns (because of the costs and constraints of some investors borrowing on their own account.) Many investment funds that specialize in providing private debt to smaller corporations would be able to use the public markets to gauge appropriate yields, diversify their risks, and hedge their risks and raise capital from investors who come to trust the market. The growth of new investors in higher yielding bonds over time will lead to a more efficient market with better pricing and more supply for emerging companies. For instance, many medium companies in the US could not raise equity or debt efficiently. And, with the development of a higher-
yielding debt market in the 1970s, entirely new corporations were born and smaller companies grew efficiently providing competition for larger companies. Sixth, a well developed and liquid corporate bond market encourages value adding innovations and their usage. It facilitates learning and also the reallocation of savings into more productive uses, and it allows corporations to make informed decisions through having more efficient and valid pricing signals. Most importantly, a legitimate market crowds out money lenders who, due to a lack of competition, are able to earn larger, abnormal returns at the expense of entrepreneurs who are striving to build their businesses and making China more competitive by reducing costs and enhancing value.

Despite all of the advantages associated with its potential future growth, China’s corporate bond market still exhibits some strange features that inhibit its proper development. This paper focuses on one in particular: the ownership distribution of Chinese corporate bonds. Earlier studies indicate that there is almost no participation from private individuals in China’s corporate bond market and corporate bonds are primarily traded and held by China’s large commercial banks (Hu 2010, Zeng 2009). Such phenomenon is drastically different from the corporate bond markets of Western economies, in which individual capital (either directly or indirectly via asset managers, insurance companies, pension funds, and money market mutual funds) makes up a significant portion of the demand side of corporate debt securities (Edwards et al 2007). The lack of private capital investment hinders the proper development of China’s corporate bond market, which consequently reduces capital accessibility of SMEs, pushing these borrowers into the shadow banking system (Yi 2009).
Chinese securities laws do not restrict participation of private capital in its capital markets (either equity or debt). It is common knowledge that China’s growing middle class are eager participants in China’s stock market (Green 2003), so why are Chinese citizens not investing in the corporate bond market? This phenomenon can of course be corrected by policy. Yet, to design and implement proper policies, it is important to understand the underlying causes first.

There are three possible explanations for this puzzle. One explanation is that Chinese citizens consciously (for utility or other considerations) choose not to participate in the bond market—in other words, perhaps because the returns are not high enough to compensate the individuals for the associated risks, there is no individual demand for corporate bonds (Hale 2007). As a result, commercial banks end up holding all of these corporate bonds as well as their direct commercial loans. Or two, although individuals would like to invest in corporate bonds, they are unable to do so because the commercial banks underwrite the entire issue and hold them for their own accounts. Only a small fraction of them, if any, reach the secondary retail market. Without liquidity and the need for market making activities, no secondary market has developed as of yet. That is, commercial banks are absorbing the entire issuance of corporate bonds to satisfy their own appetite, plan to hold the bonds to maturity, resulting in no secondary market for these securities and consequently no ownership by private individuals. Finally, the reason might be institutional. There might be barriers that block corporate bonds from entering the retail market. In other words, perhaps there is demand by individuals and no desire of hoarding by commercial banks, but institutional barriers prevent a free flow of
corporate bonds from the interbank market to the retail market, resulting in the lack of private ownership we currently observe.

This paper answers the ownership distribution puzzle by presenting a theoretical proof that Chinese citizens indeed want to hold corporate bonds if they had the ability to invest in them and they could adjust their portfolio holdings in liquid secondary markets. Hence, the current ownership distribution of corporate bonds is suboptimal from the individual’s perspective (it is also suboptimal from the issuer’s perspective in that, without touching the aggregate market, the yield on the corporate bonds might be greater than necessary to clear the market). Combining these theoretical results with some institutional details, we conclude that the reason for the lack of private ownership of corporate bonds is a combination of bank hording and institutional barriers. Assuming that enhancing citizen welfare and increasing the efficiency of the financing market for corporate projects are important priorities to the Chinese authorities, they should implement policies that allow for greater investment in corporate bonds by private individuals. Of course, we do not claim that our theoretical results (and, more importantly, their policy implications) are irrefutable. Instead, our goal is to initiate an academic discussion on this topic and hopefully our analysis and results can be useful when policy makers contemplate future policies regarding the evolution and development of China’s corporate bond market.

The remainder of this paper is organized in the following fashion. Section 2 covers relevant institutional details of China’s corporate bond markets. Section 3 sets up the model and discusses its theoretical implications. Section 4 discusses the policy implications of the theoretical results and potential shortcomings of the model. Finally,
section 5 concludes. Mathematical proofs of the theoretical conclusions are included in the appendix.

2. Institutional Background

There are four types of “corporate bonds” in China: short-term commercial paper, medium-term notes, enterprise debt, and corporate debt. The issuance and trading of each type is subject to different regulations. This section provides an overview regarding each.

2.1 Short-Term Commercial Paper and Medium-Term Notes

Both short-term commercial paper (less than 1 year maturity) and medium-term notes (maturity from 2 to 10 years) are regulated by the People’s Bank of China (PBOC) and the National Association of Financial Market Institutional Investors (NAFMII). According to Administrative Rules for the Issuance of Debt by Non-Financial Enterprises in the National Interbank Bond Market, only qualified non-financial corporate organizations are allowed to issue these types of securities. An issuer is required to register with NAFMII and registration lasts for two years. If the issuer’s credit rating deteriorates within the two years, then registration expires automatically. The size of all issuances cannot exceed 40 percent of the issuer’s net asset. The capital raised can only be used for operating activities and the particular usages have to be explicitly disclosed prior to issuance. The issue price (i.e. interest rate) is determined by market forces with no additional restrictions.

As mentioned above, the maturity for short-term commercial papers cannot exceed one year. For medium-term notes, the maturity can range from 2 to 10 years, with the most typical maturity being 5 years (Hu 2010). Once a qualified issuer registers with NAFMII, it needs to finish its first issuance within 2 months. An issuer is allowed to have
multiple issuances during the 2 year period. There are no legal requirements for collateral for these securities. The underwriter has to be a commercial bank, but an issuer can choose the bank to function as underwriter. Short term commercial papers and medium term notes are only traded in the National Interbank Bond Market (NIBM), which means that private individuals with desires to invest in these securities are blocked from doing so.

2.1 Enterprise Debt

The regulatory body for the issuance and trading of enterprise debt is the National Development and Reform Commission (NDRC). Relevant regulations include (but are not limited to) Interim Regulations on Administration of Enterprise Bonds (1993), Notice Concerning the Promotion of Enterprise Bond Market Development and Streamlining of the Bond Issuance Approval Procedures (2008), and Notice of the State-owned Assets Supervision and Administration Commission of the State Council on Issuing the Interim Measures for the Administration of Bond Issue by Central Enterprises (2008). According to the above mentioned relevant regulations, issuers are restricted to corporate organizations incorporated within China that are also state-owned enterprises. Prior to every issuance, approval from NDRC must be obtained. The size of all issuances cannot exceed 40 percent of the issuer’s net asset. The usage of capital has to be one of the following:

1. Fixed capital investment, but the total issuance size cannot exceed 60 percent of the project cost.

2. Equity purchase (either share repurchase or acquisition of another company’s stock).
3. Debt repayment.

4. Paying for operating costs, but cannot exceed more than 20 percent of issuance size.

Interest rates are set by market forces, but cannot exceed 140 percent of the deposit rate set by PBOC of same maturity length. The maturity has to fall between 3 and 20 years. The most common maturity is 10 years (Hu 2010). Once an issuance is approved, it must be completed within 2 months of the approval date. Issuers are allowed to issue non-collateralized debt, but NRDC approvals might be contingent on proper collateralization. Enterprise debt can be traded both in NIBM and on various security exchanges. Consequently, individual investors are allowed to invest in enterprise debt.

2.3 Corporate Debt

The regulatory body for the issuance and trading of corporate debt is the China Securities Regulatory Commission (CSRC). According to CSRC Pilot Rules on the Issuance of Corporate Bonds (2007), all companies publicly traded on the Shanghai Stock Exchange and Shenzhen Stock Exchange are allowed to issue corporate debt. In addition, any Chinese company with stock that trades in offshore stock exchanges is also eligible to issue corporate debt. Prior to issuance, approval from the CSRC must be obtained. The usual issue size upper bound applies, but capital usage is not restricted as long as the company’s board approves it. There is no legal upper bound on the interest rate, which is determined via a price discovery process.

Maturity has to be from 3 to 10 years. The most common maturity is 5 years (Hu 2010). One approval can be used for multiple issuances. However, the first issuance has to be completed within 6 months of the approval date and has to be at least 50 percent of
all issuances. Following issuances must be completed within 24 months of the approval date. Corporate debts are not legally required to be collateralized. Finally, corporate debts are traded on security exchanges. Hence, private individuals are allowed to invest in corporate debts.

3. The Model

3.1 Basic Setup

This is a one period model. At the beginning of the period, the agent makes his investment decisions. At the end of the period, returns are realized. The agent is assumed to maximize expected utility when making investment decisions. He is assumed to be strictly risk averse. His utility function $u(\cdot)$ is assumed to be continuously differentiable and $\lim_{x \to 0} u'(x) = \infty$.

There are three assets available: a risk free asset, risky debt, and risky stock. The risky debt and risky stock are assumed to be issued by the same company.

At the end of the period, the risk-free asset generates a net return of $\overline{r}$ with probability 1.

The company has a probability $p$ of becoming bankrupt at the end of the period.

In the event of no bankruptcy, the risky debt generates a gross return $1+i$ and the stock generates a gross return of $\frac{S_1}{S_0}$, in which $S_0$ is the initial price of the stock at the beginning of the period when investment decisions are made and $S_1$ the end-of-period price unknown at the beginning of the period. Furthermore, in the event of no
bankruptcy, \( S_i \) is a random variable with a cumulative density function (CDF) of \( F(S_i) \) over the range of \((0, \infty)\).

In the event of bankruptcy, the risky debt generates a gross return of \( \delta(1+i) \)
where \( \delta \in [0,1] \) is the recovering factor of the risky debt under bankruptcy. When making the investment decision at the beginning of the period, \( \delta \) is a random variable with CDF \( F(\delta) \). The stock generates a gross return of zero (i.e. \( S_i = 0 \)).

Without lost of generality, assume that the agent has 1 dollar to invest at the beginning of the period. His choices involve deciding what fraction of the dollar to invest in each asset. Staying consistent with current Chinese investment regulations, short selling of stock are prohibited. We also assume that the agent cannot borrow additional money to invest.

To be more precise, let \( \alpha \) be the fraction the agent invests in stock and \( \beta \) be fraction the agent invests in the risky bond, then both \( \alpha \) and \( \beta \) have to be weakly between 0 and 1 and sum up to weakly less than 1.

In addition, under option theory, it can be shown that the expected return of the stock is strictly greater than the expected return of the risky debt and the expected return of the risky debt is strictly greater than the risk-free return (Merton 1974). Hence, we use this result as an assumption here.

To recap, the exogenous factors are \( p, F(S_i), F(\delta), i, \) and \( \bar{T} \).

The choice variables are \( \alpha \) and \( \beta \).
The agent’s problem is taking the exogenous factors as given,

\[
\max_{\alpha, \beta} (1 - p) \int u \left( \frac{S}{S_0} + \beta (1+i) + (1-\alpha - \beta)(1+\delta) \right) d(F(S_i)) + p \int u \left( \delta \beta (1+i) + (1-\alpha - \beta)(1+\delta) \right) d(F(\delta))
\]

subject to \(\alpha \in [0,1], \beta \in [0,1],\) and \(\alpha + \beta \leq 1.\)

Notice that the first component in the objective function is the agent’s expected return in event of no bankruptcy while the second component is the expected return in the event of bankruptcy.

### 3.2 Theoretical Results

The main result is that not holding any risky bond (\(\beta = 0\)) is suboptimal from the agent’s perspective. This result is proven in three steps, making up the three propositions in this subsection. First, we prove that when there is no risky debt (i.e. there is only stock and risk free asset), the optimal fraction to invest in stock is strictly between zero and one (\(\alpha^* \in (0,1)\)). Second, we show that when risky debt is available, the optimal fraction to invest in stock does not change. Finally, we show that the agent can strictly increase his utility by decreasing his investment in the risk free asset and invest that incremental fraction in the risky debt, hence proving the claim that \(\beta = 0\) is suboptimal.

**Proposition 1:** When there is no risky debt available, the optimal fraction to invest in stock is strictly between zero and one (\(\alpha^* \in (0,1)\)).

**Proof:** See appendix.

The result of Proposition 1 is unsurprising and was shown by previous scholars (cite reference). In practice, the heavy participation in the stock market from China’s middle class is consistent with Proposition 1.
When risky debt becomes available, for the agent to invest in the risky debt, he needs to do at least one of the following two things:

1. Decrease his investment in stock.
2. Decrease his investment in risk-free debt.

**Proposition 2:** When risky debt is available, the optimal fraction to invest in stock remains the same. In other words, $\alpha^*$ does not change.

**Proof:** See appendix.

According to Proposition 2, if it is optimal for the agent to invest in risky debt, then he has to decrease his investment in the risk free asset.

**Proposition 3:** When risky debt is available, not holding any risky debt ($\beta = 0$) is suboptimal to the agent.

**Proof:** See appendix.

### 4. Policy Implications

Proposition 3 essentially proves that Chinese investors (especially those who already invest in the stock market) indeed want to invest in corporate bonds. Hence, the current phenomenon of essentially zero ownership of corporate bonds by private citizens is suboptimal and should be corrected by government policy.

In light of the regulations discussed in section 2, it is very difficult for small-and-medium-size enterprises to issue corporate bonds in China. To be clear, this difficulty is not generated by the market in the sense that investors demand too high of an interest rate that prevents the SMEs from issuing debt. In fact, anecdotal evidence suggests that SMEs are willing to pay extremely high interest rates (ranging from 30 to 50 percent per annum) to raise debt capital. The existence of micro financing firms, which makes up the supply
side of China’s shadow banking system, is convincing evidence that there is both supply and demand for corporate debt. Instead, many potential issuers simply cannot obtain approval from the regulatory authorities. To become a qualified issuer, company needs to meet certain risk benchmarks before it can be allowed to raise capital via corporate bond issuance. Consequently, it is often the case that companies that can qualify to issue debt in the corporate bond market do not have the financial needs, and the small amount of debt they end up issuing are quickly absorbed by the commercial banks as medium to long term assets.

On the other hand, the companies that actually have the financial need and would normally make up the supply side of the corporate bond market cannot obtain approval to participate in the corporate bond market. This result contradicts earlier conclusions published by the Federal Reserve Bank of San Francisco, which claims that there is “a lack of incentives for [Chinese] firms to issue bonds” (Hale 2007). The inability for SMEs with financing need to access the debt capital market is arguably the reason for the rapid growth of micro financing firms since its legalization in China. The micro financing firms serves as an alternative channel to connect the SMEs who are blocked from the official corporate bond market and investors seeking to invest in risky debt. The problem is that the practice methods of micro finance firms are not regulated and it increases an extra layer of risk to both investors and the SMEs, but this topic will be reserved for another paper.

The main (and also official) reason behind these stringent requirements is to protect investors from risky issuers (Chen et al 2011). Nonetheless, as long as the risks are properly disclosed and understood, there is no reason to block risky issuers from the
bond market because investors would recognize the risk and hence demand an interest rate sufficiently high to compensate them for the risk. According to our results, denying participation to risky issuers might actually reduce investor welfare.

In light of the above situation, regulators should gradually loosen the participating benchmarks faced by companies and allow the market to determine these companies’ cost of borrowing. Ultimately, instead of blocking these companies from issuing debt in the official bond market, and thus forcing them into the less regulated realm of micro financing, the policy makers should allow these companies to participate in the official bond market under the condition of proper risk disclosures. Doing so would not only satisfy these companies’ financing needs, but more importantly would enhance investor welfare.

Of course, our theoretical model assumes that investors are aware of the risk profile of the company. This is implicitly assumed in our model by assuming that the agent knows both the probability of bankruptcy and the CDF of the recovery factor in the event of bankruptcy. Hence, it is essential that the regulators pass policies that ensure proper risk disclosure by the companies that want to participate in the debt market. One can argue that given China is an emerging market, the average Chinese investor might not be very sophisticated, so even with all the proper risk disclosure, investors would not bother processing these information. We pass no judgment here on the sophistication level of Chinese investors, but even if such claims are true, perhaps the regulators should focus their energy on setting up risk evaluating entities (such as rating agencies) that can help investors better understand these risks instead of shielding the investors from risks
all together. Finally, the establishment and enforcement of bankruptcy laws are also essential to the applicability of our results.

All the above policies should increase the liquidity of the secondary bond market and hence give investors greater accessibility to corporate bonds. In addition, perhaps policy makers should also contemplate the possibility of allowing individual investors to hold short term commercial papers and medium term notes, which are currently only available to commercial banks. Given that these securities are issued by higher quality companies, the regulators can worry less about exposing individual investors to excessive risk.

Another reason voiced by the proponents of the current restrictive participation benchmarks is that, if the bond market grows rapidly, it might attract capital from the stock market (Sun and Tong 2000). If such a transition happens too quickly, it might lead to tumbling stock prices. Our results should mitigate this concern. According to Proposition 2, when risky bonds become available, the agent would not reduce his investment in the stock market. Hence, we do not expect significant disinvestment in the stock market as the bond market grows. In fact, our results suggest that investors are more likely to decrease their holding in risk free asset (i.e. decrease their savings) in order to invest in risky corporate bonds. Perhaps encouraging individual investment in corporate bonds would lower China’s high savings rate and possibly decrease China’s staggering trade surplus (McKinnon and Schnabl 2011).

5. Conclusion

Combining a simple theoretical model with some institutional details, we show that the lack of investment in China’s corporate bonds do not reflect a lack of investor
demand. Instead, it is most likely caused by a combination of institutional barriers and commercial bank hording. This suboptimal phenomenon can be modified via appropriate policy actions that include, but not limited to, lowering barriers to entry to the bond market faced by potential issuers, establishing proper information/risk disclosure requirements, increasing secondary market liquidity, and increasing the enforceability of bankruptcy laws.

Encouraging the development of China’s corporate bond market makes smaller companies more competitive by granting them capital accessibility, which would in turn stimulate growth in the domestic economy (Hasan et al 2009). Many more entrepreneurs will have access to capital than previously. In addition, opening additional investment channels for savers (corporate bonds) allows them to buy assets that are less risky than investing in tangibles such as real estate. This reduces the pressure on China’s real estate markets and mitigates the effects of bubbles in these markets. As a result, more channels allow the central banks to concentrate more on inflation targeting and less on worrying about asset bubbles in undertaking monetary policy, reserve setting and interest rate policy.
Appendix

Proof of Proposition 1:

When there is no risky debt available, the agent’s problem becomes the following

\[ \max_{\alpha} (1 - p) \int_0^\infty u \left( \alpha \frac{S}{S_0} + (1 - \alpha)(1 + \bar{T}) \right) d \left( F(S_i) \right) + pu((1 - \alpha)(1 + \bar{T})) \]

subject to \( \alpha \in [0,1] \).

\[ U \equiv (1 - p) \int_0^\infty u \left( \alpha \frac{S}{S_0} + (1 - \alpha)(1 + \bar{T}) \right) d \left( F(S_i) \right) + pu((1 - \alpha)(1 + \bar{T})) \]

Taking first order condition of the objective function with respect to \( \alpha \)

\[ \frac{\partial U}{\partial \alpha} = (1 - p) \int_0^\infty u' \left( \alpha \frac{S}{S_0} + (1 - \alpha)(1 + \bar{T}) \right) \left( \frac{S_i}{S_0} - (1 + \bar{T}) \right) d \left( F(S_i) \right) - pu'((1 - \alpha)(1 + \bar{T}))(1 + \bar{T}) \]

\[ \frac{\partial U}{\partial \alpha}_{|_{\alpha=0}} = (1 - p)u'(1 + \bar{T}) \left( \frac{E[S]}{S_0} - (1 + \bar{T}) \right) - pu'(1 + \bar{T})(1 + \bar{T}) \]

Hence, \( \frac{\partial U}{\partial \alpha}_{|_{\alpha=0}} > 0 \) if and only if \( \frac{E[S]}{S_0} - (1 + \bar{T}) > p \frac{E[S]}{S_0} \), or equivalently

\[ (1 - p) \frac{E[S]}{S_0} > (1 + \bar{T}) \]

The left hand side of (1.4) is exactly the expected payoff of the stock. Given it is assumed in Section 2.1 that the expected payoff of the stock is strictly greater than the risk free payoff, we know that \( \frac{\partial U}{\partial \alpha}_{|_{\alpha=0}} > 0 \).

\[ \frac{\partial U}{\partial \alpha}_{|_{\alpha=1}} = (1 - p) \int_0^\infty u' \left( \frac{S}{S_0} \right) \left( \frac{S_i}{S_0} - (1 + \bar{T}) \right) d \left( F(S_i) \right) - pu'(0)(1 + \bar{T}) < 0 \]

The second derivative of the objective function with respect to \( \alpha \) is
\[
\frac{\partial^2 U}{\partial \alpha^2} = (1-p) \int_{-\infty}^{\infty} u' \left( \alpha \frac{S_t}{S_0} + (1-\alpha)(1+\bar{\tau}) \right) \left( \frac{S_t}{S_0} - (1+\bar{\tau}) \right)^2 d\left( F(S_t) \right) + pu'\left((1-\alpha)(1+\bar{\tau})\right)(1+\bar{\tau})^2
\]

Because the agent is assumed to be strictly risk averse, \( \frac{\partial^2 U}{\partial \alpha^2} \) is the sum of two strictly negative numbers and hence strictly less than zero. Consequently, \( \frac{\partial U}{\partial \alpha} \) is continuous and strictly decreasing in \( \alpha \). Given that it has already be proven that \( \frac{\partial U}{\partial \alpha} \bigg|_{\alpha=0} > 0 \) and \( \frac{\partial U}{\partial \alpha} \bigg|_{\alpha=1} < 0 \), by the Continuous Mapping Theorem, there exists a unique \( \alpha^* \) strictly between 0 and 1 such that \( \frac{\partial U}{\partial \alpha} \bigg|_{\alpha=\alpha^*} = 0 \).

QED

**Proof of Proposition 2:**

\( \hat{U} = (1-p) \int_{-\infty}^{\infty} u' \left( \alpha^* - \varepsilon \right) \frac{S_t}{S_0} + \varepsilon (1+i) + (1-\alpha^*)(1+\bar{\tau}) \right) d\left( F(S_t) \right) + p \int_{0}^{1} u' \left( \varepsilon (1+i) + (1-\alpha^*)(1+\bar{\tau}) \right) d\left( F(\delta) \right) \]

where \( \varepsilon > 0 \) and arbitrarily small.

When risky debt is available, the agent will not change his investment fraction in stock if and only if

\( (2.1) \)

\[
\frac{\partial \hat{U}}{\partial \varepsilon} \bigg|_{\varepsilon=0} < 0
\]

\[
(1-p) \int_{-\infty}^{\infty} u' \left( \alpha^* \frac{S_t}{S_0} + (1-\alpha^*)(1+\bar{\tau}) \right) d\left( F(S_t) \right) + pu'\left((1-\alpha^*)(1+\bar{\tau})\right)(1+\bar{\tau})^2 E[\delta](1+i)
\]

\[< (1-p) u'\left((1-\alpha^*)(1+\bar{\tau})\right) \left[ -\frac{E[S_t]}{S_0} + 1+i \right] + pu'\left((1-\alpha^*)(1+\bar{\tau})\right) E[\delta](1+i)
\]

The right hand side of the inequality in (2.1) is strictly less than zero if and only if

(2.2) \( (1-p) \left[ -\frac{E[S_t]}{S_0} + 1+i \right] + pE[\delta](1+i) < 0 \)
The left hand side of (2.3) is the expected return of the risky debt and the right hand side is expected return of the stock. Hence, inequality (2.3) is satisfied.

QED

Proof of Proposition 3:

According to Proposition 2, the agent would not decrease his investment share in stock. Hence, the only way for him to invest a strictly positive amount in the risky debt is by decreasing his investment in the risk free asset.

\[ \tilde{U} = (1-p) \int_u^\infty u\left(\alpha^* \frac{S}{S_0} + (1-\alpha^*)(1+i)\right)(i-\bar{\gamma})d(F(S_i)) + p\int_u^\infty u\left(\delta \bar{\gamma} + (1-\alpha^*)(1+i)\right)d(F(\delta)) \]

To show that \( \beta = 0 \) is suboptimal, we need to show that \( \frac{\partial \tilde{U}}{\partial \bar{\gamma}} \bigg|_{\bar{\gamma}=0} > 0 \).

\[ \frac{\partial \tilde{U}}{\partial \bar{\gamma}} \bigg|_{\bar{\gamma}=0} = (1-p)\int_u^\infty u'\left(\alpha^* \frac{S}{S_0} + (1-\alpha^*)(1+i)\right)(i-\bar{\gamma})d(F(S_i)) + pu'\left((1-\alpha^*)(1+i)\right)(E[\delta](1+i) - (1+\bar{\gamma})) \]

\[ = (1-p)\int_u^\infty u'\left(\alpha^* \frac{S}{S_0} + (1-\alpha^*)(1+i)\right)(i-\bar{\gamma})d(F(S_i)) + pu'\left((1-\alpha^*)(1+i)\right)d(F(S_i)) \]

\[ > (1-p)\int_u^\infty u'\left(\alpha^* \frac{S}{S_0} + (1-\alpha^*)(1+i)\right)(i-\bar{\gamma})d(F(S_i)) + p\left(E[\delta](1+i) - (1+\bar{\gamma})\right)u'\left((1-\alpha^*)(1+i)\right) \]

Expression A>0 if and only if

\[ (3.1) \quad (1-p)(i-\bar{\gamma}) + p\left(E[\delta](1+i) - (1+\bar{\gamma})\right) > 0 \]

\[ (3.2) \quad \Leftrightarrow (1-p)(1+i) + pE[\delta](1+i) > (1+\bar{\gamma}) \]

The left hand side of inequality (3.2) is the expected return of the risky debt. Hence, the inequality is satisfied.

QED
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