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Coalition-Enhancing Fiscal Policies in an Open Economy: A CES Framework of Gale’s Transfer Paradox

by

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Abstract

The motivation of our paper comes from David Gale’s seminal work in 1974. He constructed an example of the “transfer paradox” based on three Leontief functions. The transfer paradox is that when there is a set of agents in the home country and that the home country is trading with other countries, then certain public lump-sum tax transfer plans could make all agents in the home country better off. Our contributions are as follows. First, we show that such an example can be constructed with three smooth CES utility functions. Secondly, we establish the three crucial conditions for the existence of the transfer paradox: (1) The donor (a taxpayer) has stronger preferences for the foreign good than the recipient; (2) The donor is ex-ante wealthier than the recipient; (3) The elasticity of substitution of the foreign country’s preference is strictly less than one.

Keywords: Transfer paradox, Open economy, Fiscal policy, CES utility function.

JEL Classification No.: F41, F42, H30.

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1 Introduction

Coalition of certainty groups are known to affect market prices under a general equilibrium setting. These price changes affect agents’ real wealth and consequently, their utility values. As early as 1939, Samuelson, while studying the benefit of government intervention in international trade, suggested the possibility of lump-sum transfers that are Pareto-improving for all agents within a coalition in an economy. Grandmont and McFadden (1972) prove the result for models with production. Assuming that there is a set of agents in our home country and that the home country is trading with other countries, then certain public lump-sum tax transfer plans could make all agents in the home country better off.

Despite this early fundamental insight, there is a paucity of examples that demonstrate this phenomenon. Gale (1974) led the effort, later followed by theoretical works by Guesnerie and Laffont (1978), Bhagwati, Brescher, and Hatta (1983, 1984), Yano (1983), Jones (1984), and others. However, Gale confined his example to non-smooth preferences in general equilibrium frameworks. Gale utilized three Leontief functions (two for the home country and one for the foreign country) to show Samuelson’s insight. He pointed out that there had been several unsuccessful attempts to construct examples with smooth preferences. Léonard and Manning (1983) partially addressed this challenge by showing an example in general equilibrium with two Cobb-Douglas utility functions with specified preference parameter values in the home country and one smooth demand function in the foreign country.

Our paper builds on these works by constructing a theoretical framework with three smooth CES utility functions, which yields results that relate pref-

\[1\] Bhagwati, Brescher, and Hatta (1983, 1984) and Yano (1983) utilized three-country frameworks. Gale (1974) studied the setting of two home country agents and one foreign country representative agent. These are two different interpretations of the three-agent transfer paradox. Mathematically, these two structures are no different. As Bhagwati, Brescher, and Hatta (1983) mention, the results from their setting of three independent countries can also be applied to Gale (1974)’s framework.
ferences, fiscal transfer, and welfare that previous examples could not reflect. More specifically, for the home country with two representative consumers, we first maintain the use of two Cobb-Douglas functions but relax the preference parameters, allowing preference diversity between the two types of consumers to generate the incentive for utilizing a public lump-sum tax transfer plan. In addition, our example adopts a more general utility specification for a representative consumer in the foreign country, which shows how the foreign consumer’s elasticity of substitution plays a crucial role in determining the magnitude of the fiscal transfer’s effect. Most importantly, we derive a threshold value for the foreign consumer’s elasticity of substitution such that coalition could only succeed below this value. The mechanism that allows wealth transfer from the foreign to the domestic country is the decrease in the price of the foreign good as a result of domestic fiscal policy. This result is similar in spirit to those derived from the literature on immiserizing growth (Bhagwati 1958, 1968) and endowment manipulation (Aumann and Peleg 1974, Postlewaite 1973). The idea is that real wealth of the endowment loser is affected by both endowment change and price change that work in opposite directions, and the latter effect will outweigh the former, generating a counterintuitive real wealth gain.\(^2\)

Our paper proceeds as follows. First, we construct a model which can be analyzed in CES utility functions. By using two Cobb-Douglas utility functions for the home country and one Leontief function for the foreign country, we obtain closed form solutions for all equilibrium allocations and have utilities expressed in terms of lump-sum transfers. We derive preference parameter restrictions that allow coalitions as described above to succeed. Secondly, we demonstrate Samuelson’s insight with three perfectly smooth CES utility functions. Due to the Cobb-Douglas forms of the two home consumers’ preferences, we can use one aggregate utility form that al-

\(^2\) Analogously, Johnson (1953), Kenan and Riezman (1988), and Syropoulos (2002) have also shown that tariffs can be used as a tool for decreasing the value of the foreign good in order to increase welfare in the home country.
together incorporates their preferences. This would lead to equivalent price changes as if there were two different consumers in the economy. The lump-sum tax affect the aggregator’s preferences, resulting in price changes, and subsequently, wealth transfer from the foreign to the home country. Given these price changes, we show that the foreign country’s elasticity of substitution is a critical determinant of the magnitude of the wealth transfer. A more complementary-oriented elasticity of substitution implies that prices are more sensitive to endowment redistribution, and therefore coalition in the home country is more likely to succeed. Previously, Bhagwati, Brecher and Hatta (1983, 1984) and Yano (1983) have also demonstrated that lower elasticity of substitution increases the likelihood of the transfer paradox. However, in contrast to these works, our model presents parametric examples with utility functions, and more importantly, with CES preferences, and as a result we are able to derive a precise relationship between the elasticity of substitution and the possibility of the transfer paradox. Lastly, we examine the distributional consequences of this wealth transfer on the home country’s two consumers. The agent who bears the tax burden will also receive wealth compensation through the international trade channel. We show that a coalition is more likely to succeed if the consumer whose wealth is taxed has greater ex-ante endowment.

Section 2 introduces the theoretical framework with CES utility functions. Section 3 analyzes the case in which the foreign country has Leontif preferences. Section 4 extends the framework to three smooth CES preferences. Section 5 concludes.

2 THEORETICAL FRAMEWORK

There are two agents, denoted by the subscripts $A$ and $B$, in the home country, and one representative agent, $C$, in the foreign country. Agents in each country consume goods $x$ and $y$, which denote the goods produced by
the domestic and foreign country, respectively. The preferences of consumers in the home country are given by the utility specifications

\[ U_A(x_A, y_A) = x_A^\alpha y_A^{1-\alpha}, \]
\[ U_B(x_B, y_B) = x_B^\beta y_B^{1-\beta} \]

where \( \alpha > \beta \) and \( \alpha, \beta \in (0, 1) \)

In other words, agent A’s relative preference for the domestic good is higher than that of agent B.

The preference of agent C in the foreign country is given by

\[ U_C(x_C, y_C) = \begin{cases} 
\left( \frac{x_C^{\varepsilon+1}}{y_C^{\varepsilon+1}} \right)^{\frac{\varepsilon}{\varepsilon+1}} & \text{if } \varepsilon \in (0, \infty)/\{1\} \\
x_C y_C & \text{if } \varepsilon = 1 \\
\min(x_C, y_C) & \text{if } \varepsilon = 0 
\end{cases} \]

where \( \varepsilon \) represents the elasticity of substitution of the foreign country.

The endowment of each agent is given by

\[ \omega_A = (\theta, 0), \omega_B = (1-\theta, 0) \text{ and } \omega_C = (0, 1). \]

We can see that the home country and foreign country are specialized in goods \( x \) and \( y \), respectively, but they both have the same amount of aggregate endowment, 1. \( \theta \in (0, 1) \) represents the initial endowment distribution share in the domestic country. If the utility functions of the two domestic consumers are both strictly increasing in \( \theta \), there would exist a lump-sum transfer plan in the domestic country that makes the two consumers better off. Thus, we will derive \( \theta \) for which agents A and B’s utility are both strictly increasing. If there exists such a value or range of \( \theta \), we can conclude that a Pareto-improving coalition in the home country is possible. Formally, we define coalition success as the following:
**Definition 1** A coalition succeeds if there exists a set of endowment transfer from agent B to A such that \( \frac{dU_A}{d\theta} > 0 \) and \( \frac{dU_B}{d\theta} > 0 \).

Definition 1 sets the case where both the donor, agent B, and the recipient, agent A, can be better off through endowment transfers. Mathematically, if such a case exists, the endowment transfer in the opposite direction, where agent A is the donor while agent B is the recipient, would make both of them worse off. It follows mathematically that since the utility change through transfer from B to A is \( \frac{dU_h}{d\theta} \), where \( h \in \{A, B, C\} \), then that from A to B is \( \frac{dU_h}{d(\theta^{-1})} \), which is the same as \(-\frac{dU_h}{d\theta}\). This is the case of another "transfer paradox," where even the recipient becomes worse off along with the donor.

### 3 The Case Where the Foreign Country Has Leontief Preferences

In this section, we will show that there exists a closed form solution in the special case in which the foreign agent has Leontief preferences. From the closed form solution, we derive important intuitions relating to the role of ex-ante wealth distribution and preference diversity in increasing the likelihood of Pareto-improving coalition.

We will use an aggregator’s utility function to represent the two Cobb-Douglas utility functions of the home consumers, which is useful to understand the changes in domestic aggregate demand for the foreign good after the tax policy is implemented. With the home country represented by the aggregator and the foreign country, we can derive the equilibrium prices for the domestic and foreign good after trade. In particular, we will examine how the price of the foreign good, denoting the domestic good as a numeraire, is affected by the lump-sum tax transfer.
The utility function of the aggregator of $A$ and $B$ is derived as\(^3\)

\[
U_{AG}(x_{AG}, y_{AG}) = x_{AG}^\eta y_{AG}^{1-\eta}
\] (1)

where $\eta = \alpha \theta + \beta (1 - \theta)$

and $\omega_{AG} = (1, 0)$ (2)

The weighting factor, $\eta$, is increasing in $\theta$. In other words, the more wealth is endowed to consumer $A$, the more the aggregator prefers the domestic good.

The demand function of the foreign good can be derived from equations (1) and (2):

\[
D_{AG}^y(q) = (1 - \eta) \frac{1}{q}
\] (3)

where $q$ is the price of the foreign good in terms of the home good.

It follows that demand is decreasing in $\theta$

\[
\frac{\partial D_{AG}^y(q)}{\partial \theta} = - (\alpha - \beta) \frac{1}{q} < 0
\] (4)

As endowment wealth is transferred from consumer $A$ to consumer $B$,

\[^3\text{Given } p = (1, q), \text{ the home country aggregate demand is the sum of two agents’ individual demands:} \]

\[
(x_A + x_B, y_A + y_B) = \left(\alpha \theta + \beta (1 - \theta), \frac{(1-\alpha) \theta}{q} + \frac{(1-\beta) (1-\theta)}{q}\right).
\]

Defining $\eta$ as $\alpha \theta + \beta (1 - \theta)$, the aggregate demand is

\[
(x_A + x_B, y_A + y_B) = \left(\eta, \frac{1-\eta}{q}\right).
\]

The demand function can be derived assuming that the utility and the endowment are of the following

\[
U_{AG}(x_{AG}, y_{AG}) = x_{AG}^\eta y_{AG}^{1-\eta}
\]

and $\omega_{AG} = (1, 0)$
the aggregator’s demand for the foreign good is decreasing. From (4), the impact of tax policy on domestic demand for the foreign good is a function of \((\alpha - \beta)\), the difference in the two home country consumers’ preferences.

We now derive the foreign country’s supply from its utility function, which is

\[ U_C(x_C, y_C) = \left( \frac{x_C^{\frac{1}{\alpha}}}{\theta} + \frac{y_C^{\frac{1}{\beta}}}{\theta} \right)^{\frac{1}{\xi}} \]

Subsequently, the supply function for the foreign good can be derived as\(^4\)

\[ S^y(q) = \frac{1}{1 + q^{1-\xi}} \quad (5) \]

The price of the foreign good, \(q\), can be derived from equating (3) and (5).

In this section, we focus on the case where the foreign country’s preferences are perfectly complementary-oriented (i.e., \(\varepsilon = 0\).) In the next section, we will address the general case where \(\varepsilon > 0\). Where \(\varepsilon = 0\), the price is computed as

\[ q = \frac{(1 - \alpha) \theta + (1 - \beta)(1 - \theta)}{\alpha \theta + \beta (1 - \theta)} \quad (6) \]

which is necessary to prove the following Proposition 1. We can also see from equation (6) that \(q\) is decreasing in \(\theta\) if \(\alpha > \beta\).

**Proposition 1** If \(\varepsilon = 0\) and \(\alpha > 2\beta\), there exists \(\overline{\theta} \in (0, 1)\) for which

\[ \frac{dU_A}{d\theta} > 0 \text{ and } \frac{dU_B}{d\theta} > 0, \text{ where } 0 < \theta < \overline{\theta} \]

\(^4\)The foreign good supply is the same as the foreign country’s endowment minus foreign country’s demand for foreign good:

\[ S^y(q) = 1 - \frac{1}{q^{\varepsilon-1} + 1} = \frac{1}{1 + q^{1-\varepsilon}} \]
and such that
\[
\bar{\theta} = \frac{1}{(\alpha - \beta)} \left[ \frac{1}{2} (2 - 3\beta) - \sqrt{\frac{1}{4} (2 - 3\beta)^2 - (\alpha - 2\beta) (1 - \beta)} \right].
\] (7)

Proof: Please see Appendix A

Remark 1 As mentioned at the end of Section 2, if Pareto-improving fiscal transfers from B to A exist for a certain range, then for the same range there exist the Pareto-disimproving fiscal transfers from A to B. The same reasoning is also applied to Propositions 2 and 3.

Proposition 1 implies that \( \alpha > 2\beta \) is the key necessary condition for the coalition to succeed even for very low values of \( \varepsilon \) - sufficient difference in the two home consumers' relative tastes in the domestic and foreign good is required for a successful coalition.

Second, Proposition 1 also implies that \( \theta \) should be sufficiently low. The range of \( \theta \) for which coalition succeeds is \( 0 < \theta < \bar{\theta} \), rather than \( \bar{\theta} < \theta < 1 \). In other words, the ex-ante wealth of consumer B should be sufficiently high relative to consumer A. This can be understood by the marginal wealth gain through transfer, \( \frac{1}{\lambda_h} dU_h d\theta \), where \( \lambda_h \) is consumer \( h \)'s marginal utility of wealth.\(^5\) Since the unit of \( \frac{1}{\lambda_h} dU_h d\theta \) is the same as \( \text{"wealth"} \) for all \( h \in \{A, B, C\} \), \( \frac{1}{\lambda_A} dU_A d\theta + \frac{1}{\lambda_B} dU_B d\theta \) represents the total marginal wealth with respect to transfer in the home country. \( \frac{1}{\lambda_A} dU_A d\theta + \frac{1}{\lambda_B} dU_B d\theta \) is computed as

\[
\frac{1}{\lambda_A} \frac{dU_A}{d\theta} + \frac{1}{\lambda_B} \frac{dU_B}{d\theta} = \left( -\frac{dq}{d\theta} \xi^A \right) + \left( -\frac{dq}{d\theta} \xi^B \right)
\]

Price effect for A  Price effect for B

\(^5\) To the best of our knowledge, the normalized ordinal utility term, which is \( \frac{1}{\lambda_h} dU_h d\theta \) here, in welfare analysis was first suggested by Donsimoni and Polemarchakis (1994). They show that if markets are complete, any endowment transfer among agents does not affect the normalized aggregate ordinal utility, which is \( \frac{1}{\lambda_A} \frac{dU_A}{d\theta} + \frac{1}{\lambda_B} \frac{dU_B}{d\theta} + \frac{1}{\lambda_C} \frac{dU_C}{d\theta} = 0 \). Therefore, the domestic wealth gain, \( \frac{1}{\lambda_A} \frac{dU_A}{d\theta} + \frac{1}{\lambda_B} \frac{dU_B}{d\theta} \), is the same as the foreign wealth loss, \( \frac{1}{\lambda_C} \frac{dU_C}{d\theta} \).
where $z^y_A$ and $z^y_B$ are agent A and B’s excess demands for the foreign good. $-\frac{dq}{d\theta} z^y_A$ and $-\frac{dq}{d\theta} z^y_B$ represent gains of agents A and B due to the transfer. The coalition is more likely to succeed when $z^y_B$ is high for any given price change, $-\frac{dq}{d\theta}$. We will show that $z^y_B$ is higher if agent B’s initial wealth, $1 - \theta$, and preference for the foreign good, $1 - \beta$, is high. Since neither agent in the home country has any endowment of the foreign good, their excess demands equal their actual demands. That is,

$$z^y_A = \frac{(1 - \alpha) \theta}{q} \quad \text{and} \quad z^y_B = \frac{(1 - \beta) (1 - \theta)}{q}$$

We can see that $z^y_A$ and $z^y_B$ are linear functions of their ex-ante endowment shares, $\theta$ and $1 - \theta$. For small value of $\theta$, agent B’s wealth transfer due to any given price change will be larger. Therefore, smaller value of $\theta$ increases the likelihood of coalition success.

Figure 1 shows a numerical simulation of the range of $\theta$ for which the utilities of consumers A and B are both strictly increasing. In this example, $\alpha = 3/4$, $\beta = 1/4$ and $\varepsilon = 0$. We can see that coalition would succeed for $0 < \theta < 0.34862$, which can be computed by equation (5). This numerical example also clearly shows that the range of $\theta$ for which coalition succeeds
is $0 < \theta < \bar{\theta}$, where the donor is relatively wealthier than the recipient compared to the range $\bar{\theta} < \theta < 1$.

4 Elasticity of Substitution and Smooth CES Preferences

In this section, we extend the previous framework by introducing three smooth CES preferences. We then demonstrate that under this general framework, there exists a critical value for the elasticity of substitution of the foreign agent’s preferences below which the coalition can succeed.

From equation (5), the price elasticity of supply can be derived as:

$$E_s = \frac{\partial S_y(q)/S_y(q)}{\partial q/q} = (\varepsilon - 1) \frac{1}{q^{\varepsilon-1} + 1} \quad (9)$$

When $\varepsilon = 1$ (Cobb-Douglas), the elasticity of supply is zero, which means that supply is perfectly inelastic. Figure 2 shows that the shape of the supply curve will differ with varying levels of $\varepsilon$. Figure 3 shows that a lower degree of elasticity of substitution for the foreign good supply will cause the foreign
good price to decrease more after any given demand shift.

The figures highlight the crucial role of the elasticity of substitution in increasing the likelihood of the transfer paradox, as shown formally in the following two propositions.

**Proposition 2** For any value of $0 \leq \varepsilon < 1$, there exists a continuum of values of $(\alpha, \beta, \theta) \in (0, 1)^3$ for which the coalition succeeds.

Proof: Please see Appendix B.

**Proposition 3** If $1 \leq \varepsilon < \infty$, there is no possible $(\alpha, \beta, \theta) \in (0, 1)^3$ for which the coalition succeeds.

Proof: Please see Appendix C.

Propositions 2 and 3 mean that if the foreign consumer’s preferences were more complementary-oriented, coalition would more likely succeed. The critical value of $\varepsilon$ that determines the possibility of successful coalitions is 1, above which no coalition is possible for any value of $\alpha$, $\beta$, and $\theta$. As a related point, proposition 3 also implies that we cannot create any transfer paradox examples with three Cobb-Douglas preferences (see Lemma 1 in the Appendix for detailed proof.)
5 Conclusion

We have shown a theoretical framework for coalition-enhancing fiscal transfers that considers a number of important new dimensions. The fundamental premise of our framework is that in order for the home country consumers to be better off through coalition, tax policies need to affect prices in a way that leads to effective wealth transfer to the home country. Thus, the coalition should be formed to decrease the price of the foreign good and increase the price of the domestic good. That is, coalitions seek to improve the terms of trade. Under such a coalition, the home country can achieve higher wealth by supplying the domestic product at a higher price and buying the foreign product at a lower price.

We have assumed that there are two types of agents in the home country, and they differ in their relative preferences for domestic vs foreign goods. Given this key characteristic, as the home country tax policy redistributes endowment from the consumer who has higher relative preference for the foreign good to the one with higher relative preference for the domestic good, the aggregate demand for the foreign good in the home country will decrease. Consequently, the world price of the foreign good will drop. The extent to which price changes is a function of the elasticity of substitution of the foreign country. We have derived a specific critical value of this elasticity of substitution for which coalitions could only succeed below this value.

In addition to the aggregate economic consequences on the home country, we have also asked whether the positive wealth effect through international trade outweighs the negative wealth effect from redistribution for the consumer who is taxed. If so, the coalition is more likely to succeed. The answer depends on the initial endowment level of the consumer who is taxed – higher initial endowment implies higher likelihood for coalition success.

The examples demonstrated in this paper highlight the importance of seemingly unilateral fiscal policies in an open economy setting. With sufficiently diverse preferences of consumers in the home country, fiscal policy
that simply reallocate wealth between agents in an economy can in fact induce terms of trade shocks that lead to additional implications on real wealth through the price effect. Thus, these results are consistent with and complements the more recent literature on fiscal devaluation (Farhi et al 2011), which has shown that a simultaneous tariff on imports and subsidy on exports can achieve the same allocations as nominal exchange rate depreciation. The results shown in this paper have provided an additional perspective – non-distortionary fiscal policies can also have non-trivial implications on the real exchange rate and could be viewed as an alternative to nominal exchange rate changes.

Appendix

A Proof of Proposition 1

Given $p = (1, q)$, the three agents’ demands are given by

$$(x_A, y_A) = \left( \alpha \theta, \frac{(1 - \alpha) \theta}{q} \right)$$

$$(x_B, y_B) = \left( \beta (1 - \theta), \frac{(1 - \beta) (1 - \theta)}{q} \right)$$

and

$$(x_C, y_C) = \left( \frac{q}{1 + q}, \frac{q}{1 + q} \right).$$

By market clearing conditions, we can derive the following two equations,

$$\alpha \theta + \beta (1 - \theta) + \frac{q}{1 + q} = 1 \quad (10)$$

and

$$\frac{(1 - \alpha) \theta}{q} + \frac{(1 - \beta) (1 - \theta)}{q} + \frac{q}{1 + q} = 1. \quad (11)$$
From equations (10) and (11), we can solve for $q$:

$$q = \frac{(1 - \alpha) \theta + (1 - \beta)(1 - \theta)}{\alpha \theta + \beta(1 - \theta)}$$  \hspace{2cm} (12)

Equation (12) can be directly derived from equations (3) and (5), which denote the foreign good demand and supply, respectively.

Now, we need to prove that as $\theta$ is increasing, both agents $A$ and $B$’s utility values are increasing. There are two effects determining utility values of the two agents. The first effect is the wealth effect driven by endowment transfers. This positively affects agent $A$ but negatively affects agent $B$. The second effect is the price effect from the change of the price ratio, $q$. It is clear that the second effect positively affects the two agents utility by revealed preference hypothesis. Therefore, agent $A$’s utility is surely increasing in $\theta$. The following proof focuses on the utility value change of agent $B$.

Substituting the price $q$ into agent $B$’s consumption, we get:

$$(x_B, y_B) = \left(\beta (1 - \theta), \frac{(1 - \beta)(1 - \theta)(\alpha \theta + \beta(1 - \theta))}{(1 - \alpha) \theta + (1 - \beta)(1 - \theta)}\right) .$$

Substituting consumption into the utility function of agent $B$, we get:

$$U_B = \beta \log \beta (1 - \theta) + (1 - \beta) \log \left(\frac{(1 - \beta)(1 - \theta)(\alpha \theta + \beta(1 - \theta))}{(1 - \alpha) \theta + (1 - \beta)(1 - \theta)}\right)$$

$$= \log \beta^\beta (1 - \beta)(1 - \theta) + \log (1 - \theta) + (1 - \beta) \log \frac{(\alpha \theta + \beta(1 - \theta))}{(1 - \beta) - \theta (\alpha - \beta)}$$  \hspace{2cm} (13)

For computational convenience, let’s define $k(\theta)$ as $k(\theta) = (1 - \beta) - \theta (\alpha - \beta)$. Then, $k'(\theta) = - (\alpha - \beta)$. Since $k(\theta)$ is monotonic, $0 < k(0)$, and $k(1) < 1$, it follows that $k(\theta) \in (0, 1)$.

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6 For computational convenience, we assume that agent $B$’s utility function is

$$\beta \log x_B + (1 - \beta) \log y_B$$

instead of $x_B^{\beta}y_B^{1-\beta}$ in the proofs of Propositions 1, 2 and 3.
Substituting $k(\theta)$ into equation (13),

$$U_B = \log \beta^\theta (1 - \beta)^{(1-\beta)} + \log (1 - \theta) + (1 - \beta) \log \frac{1 - k(\theta)}{k(\theta)}$$

Taking the derivative of the utility function of $A$ with respect to $\theta$, we have

$$\frac{dU_A(\theta)}{d\theta} = \frac{-1}{(1 - \theta)} + (1 - \beta) \left( \frac{-k'(\theta)}{1 - k(\theta)} - \frac{k'(\theta)}{k(\theta)} \right).$$

Endowment effect

Price effect

$$= \frac{-1}{(1 - \theta)} + \frac{- (1 - \beta) k'(\theta)}{(1 - k(\theta)) k(\theta)} \tag{14}$$

From equation (14), the necessary and sufficient condition for $\frac{dU_A(\theta)}{d\theta}$ being strictly positive is that

$$f(\theta) = -k^2(\theta) + k(\theta) + k'(\theta)(1 - \beta)(1 - \theta) < 0.$$  

The inequality is identical to

$$f(\theta) = -\{(1 - \beta) - \theta (\alpha - \beta)\}^2 + \{(1 - \beta) - \theta (\alpha - \beta)\} - (\alpha - \beta)(1 - \beta)(1 - \theta)$$

$$= -\theta^2 (\alpha - \beta)^2 + \theta (\alpha - \beta)(2 - 3\beta) + (2\beta - \alpha)(1 - \beta)$$

$f(\theta)$ is a continuous quadratic function on $(0,1)$. The following is the sufficient condition for the existence of $\bar{\theta}$, so that for any $\theta \in (0, \bar{\theta})$, $f(\theta) < 0$:

$$f(0) < 0 \quad \text{and} \quad f(1) > 0.$$

The second condition, $f(1) > 0$, is automatically satisfied by the assumption $\alpha \in (0,1)$ since

$$f(1) = - (\alpha - \beta)^2 + (\alpha - \beta)(2 - 3\beta) + (2\beta - \alpha)(1 - \beta)$$

$$= \alpha(1 - \alpha) > 0.$$
The first condition is that

\[ f(0) = (2\beta - \alpha) (1 - \beta) < 0 \quad (15) \]

The inequality (15) is satisfied where

\[ \alpha > 2\beta. \quad (16) \]

Otherwise, there is no possible coalition for any \( \theta \in (0, 1) \).

Finally, we can derive \( \bar{\theta} \in (0, 1) \) by solving the equation, \( f(\bar{\theta}) = 0 \):

\[ \bar{\theta} = \frac{1}{(\alpha - \beta)} \left[ \frac{1}{2} (2 - 3\beta) - \sqrt{\frac{1}{4} (2 - 3\beta)^2 - (\alpha - 2\beta) (1 - \beta)} \right] \]

Figure (4) helps to understand the proof.
B Proof of Proposition 2

Agent B’s demand function, given \( p = (1, q) \), is

\[
(x_B, y_B) = \left( \beta (1 - \theta), \frac{(1 - \beta)(1 - \theta)}{q} \right)
\]

Then, utility of agent B is

\[
U_B(\theta) = \beta \log \beta (1 - \theta) + (1 - \beta) \log \frac{(1 - \beta)(1 - \theta)}{q}
\]

\[
= \log (1 - \theta) - (1 - \beta) \log q + \log \beta^\beta (1 - \beta)^{1-\beta}
\]

Taking the total derivative of \( U_B(\theta) \) with respect to \( \theta \), we get

\[
\frac{dU_B(\theta)}{d\theta} = (1 - \beta) \left( -\frac{1}{q} \frac{dq}{d\theta} \right) - \frac{1}{1 - \theta}
\]

(17)

From equation (17), we can see that a higher value of \(-\frac{1}{q} \frac{dq}{d\theta}\) implies that the coalition is more likely to succeed. The price of the foreign good, \( q \), can be computed by equating the supply and demand of foreign goods. Supply and demand functions for foreign goods are given by, respectively,

\[
S(q) = \frac{1}{1 + q^{1-\epsilon}} \quad \text{and}
\]

\[
D(q, \theta) = (1 - \eta) \frac{1}{q} \quad \text{where} \quad \eta = \alpha \theta + \beta (1 - \theta)
\]

The price \( q \) is determined by the following relation:

\[
S(q) = D(q, \theta) \quad \text{and}
\]

(18)

\[\text{This relation is equivalent to the market clearing condition for foreign goods.}\]
Implicitly differentiating equation (18) with respect to $\theta$, we can obtain

$$S'(q) \frac{dq}{d\theta} = D_1(q, \theta) \frac{dq}{d\theta} + D_2(q, \theta)$$

Then, the price elasticity with respect to $\theta \left( -\frac{1}{q} \frac{dq}{d\theta} \right)$ is computed as

$$-\frac{1}{q} \frac{dq}{d\theta} = \frac{(\alpha - \beta)}{q^2 S'(q) + (1 - \eta)}$$

$$= \frac{(\alpha - \beta)}{q^2 S'(q) + (1 - \beta) - \theta (\alpha - \beta)}$$

which is decreasing in $q^2 S'(q)$. Therefore, the lower the value of $q^2 S'(q)$, the more likely that the coalition would succeed.

Substituting $-\frac{1}{q} \frac{dq}{d\theta}$ into equation (17), we can get:

$$\frac{dU_B(\theta)}{d\theta} = \frac{(1 - \beta) (\alpha - \beta)}{q^2 S'(q) + (1 - \beta) - \theta (\alpha - \beta)} - \frac{1}{1 - \theta}$$

In the above equation, $q^2 S'(q)$ is the only part that is a function of $q$. $q^2 S'(q)$ is computed as,

$$q^2 S'(q) = -\frac{(1 - \varepsilon) q^{-\varepsilon+2}}{(1 + q^{1-\varepsilon})^2}$$

which is strictly negative where $\varepsilon < 1$ for any value of $q > 0$. Finally, $\frac{dU_B(\theta)}{d\theta}$ can be rewritten as

$$\frac{dU_B(\theta)}{d\theta} = \frac{1}{\frac{q^2 S'(q)}{(1-\beta)(\alpha-\beta)} + \frac{1}{(\alpha-\beta)} - \theta \frac{1}{(1-\beta)}} - \frac{1}{1 - \theta}$$

\text{Price effect}\quad \text{Endowment effect}

As $\alpha \to 1$ and $\beta \to 0$, the price effect converges to

$$\frac{1}{q^2 S'(q) + 1 - \theta}$$
which is greater than \( \frac{1}{1-\theta} \) since \( q^2 S'(q) \) is strictly negative. Therefore, for any \( \theta \in (0, 1) \), there exist possible values of \((\alpha, \beta) \in (0, 1)^2\) for the coalition to succeed if and only if \( \varepsilon < 1 \).

C  PROOF OF PROPOSITION 3

Before the main proof, we will prove the following lemma:

Lemma 1  If the foreign country's utility function is Cobb-Douglas, i.e., \( \varepsilon = 1 \), no coalition is possible for any value of \((\alpha, \beta, \theta) \in (0, 1)^3\).

Proof of Lemma 1: If the foreign country is represented by a Cobb-Douglas utility function, i.e., \( \varepsilon = 1 \), then \( S'(q) = 0 \). (See Appendix B.) Then, \( \frac{dU_B(\theta)}{d\theta} \) is

\[
\frac{dU_B(\theta)}{d\theta} = (1-\beta) \left( \frac{\alpha-\beta}{1-\alpha\theta - \beta (1-\theta)} \right) - \frac{1}{1-\theta} \\
= \frac{(1-\beta)(\alpha-\beta)}{(1-\beta)-(\alpha-\beta)\theta} - \frac{1}{1-\theta} 
\]

(20)

For the coalition to succeed, \( \frac{dU_B(\theta)}{d\theta} \) should be strictly positive. However, we need to prove that \( \frac{dU_B(\theta)}{d\theta} \) cannot be strictly positive for any values of \((\alpha, \beta, \theta)\). From equation (20), the necessary and sufficient condition for \( \frac{dU_B(\theta)}{d\theta} \) to be strictly positive is that

\[
\frac{1}{\alpha - \beta} - \frac{1}{1-\beta} \theta > 1-\theta 
\]

(21)

Define \( t(\theta) = 1 - \frac{1}{\alpha-\beta} + \left( \frac{1}{1-\beta} - 1 \right) \theta \), the inequality (21) is equivalent to

\[
t(\theta) < 0
\]
Since $t(\theta)$ is strictly increasing in $\theta$, we only need to check one value of $\theta$: $\theta = 1$. When $\theta = 1$, $t(\theta)$ is

$$t(1) = -\frac{1}{\alpha - \beta} + \frac{1}{1 - \beta} < 0$$

since $\alpha - \beta < 1 - \beta$. ■

Remembering equation (19) from Appendix B, $\frac{dU_B(\theta)}{d\theta}$ is given by

$$\frac{dU_B(\theta)}{d\theta} = \frac{q^2S'(q)}{(1-\beta)(\alpha-\beta)} + \frac{1}{(\alpha-\beta)} - \theta \frac{1}{1-\beta}$$

Price effect

$$\frac{1}{1-\theta}$$

Endowment effect

where $q^2S'(q)$ is derived as

$$q^2S'(q) = -(1-\varepsilon)q^{-\varepsilon+2}$$

which is strictly positive if $\varepsilon > 1$. Since (1) $\frac{dU_B(\theta)}{d\theta}$ is decreasing in $q^2S'(q)$ and (2) there is no coalition where $S'(q) = 0$ (by Lemma 1), we can conclude that there is no possible coalition if $q^2S'(q) > 0$. The proof is completed by the fact that $S'(q) \geq 0$ if and only if $\varepsilon \geq 1$.

Q.E.D.

References


