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An exploration of China's mortality decline under Mao: A provincial analysis, 1950-80

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RATIONING, SPILLOVER, AND INTERLINKING IN CREDIT MARKETS: THE CASE OF RURAL PUNJAB

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A parallel market structure may exhibit extensive rationing in the regulated segment, and hence spillover of unsatisfied demand into the unregulated segment of the market. In the latter segment, the borrower can choose to bundle loan contracts with output marketing through the lender. Using data on Punjabi cultivators, econometric estimation of such a structure yields three principal findings: (i) most borrowers and non-borrowers were rationed in the regulated market; (ii) demand for credit was fairly inelastic with respect to the interest rate; (iii) a contractual provision tying credit to output marketing made informal lenders willing to advance much bigger loans.

One pana and a quarter is the lawful rate of interest per month on one hundred panas, five panas for purposes of trade, ten panas for those going through forests, twenty panas for those going by sea. For one charging or making another charge a rate beyond that, the punishment shall be the lowest fine for violence, for witnesses, each one of them, half the fine. If, however, the King is unable to ensure protection, the (judge) should take into consideration the usual practice among creditors and debtors. The Kautiliya Arthasastra (a fourth century BC work on kingship and statecraft).

1. Introduction

A salient feature of rural credit markets in less-developed countries is that formal institutions do business alongside private lenders. Whereas the latter’s operations are not effectively regulated, governments usually impose a ceiling on the rate of interest institutions may charge. Segmentation of the market for loans, with rationing in the regulated segment, is a likely outcome. Some unsatisfied demand may then spill over into the unregulated segment; and if neither institutional nor private lenders can enforce exclusive contracts, some applicants will succeed in obtaining loans from both sources. While the two segments of the market operate in parallel, there is therefore substantial scope for particular forms of interaction between them.

The set of possibilities—and complications—is enriched by the phenomenon of interlinking (Bardhan, 1980; Bell, 1988) whereby private credit contracts are bundled together with other transactions between the two parties, a practice which is closely related to the contract farming observed in some parts of the world (see Little and Watts, 1994). In commercialized areas of India, for example, it is common for some or all of the borrower’s crops to be marketed through the commission agent who provided the credit. The terms of the
contract include not only the size of the loan and the rate of interest, but also the quantity of the output to be marketed and the commission payable thereon. All contractual terms, including whether the loan is tied, are simultaneously determined as part of the package. These contracts typically are neither registered nor even witnessed by a third party, and fall under customary law.

The question of debt seniority also arises when contracts are not exclusive. Commission agents have effective debt seniority over other lenders because they are in a position to deduct principal, interest, and commission charges from the proceeds of the sale, even when their clients borrowed first from other sources. As for enforcement, commission agents usually have their shops or warehouses in the marketing centers, where they can keep an eye on their clients’ movements. It is also common for commission agents to withhold a 'normal' amount of principal and interest on the output marketed through them by farmers with whom they have no credit contracts until they are satisfied that the farmers in question had not entered into debt contracts with other commission agents, a practice made possible by the close social and business network that binds the trading community together.

We develop and estimate a model of the market for short-term loans in Punjab based on data collected in a 1980-81 survey, with special attention to four prominent institutional features of this market. First, we model the two distinct segments of the market, incorporating the fact that there was widespread borrowing from both institutional and private sources (see Table 1 in Section 4). Second, we allow for the possibility of rationing in the institutional sector and associated spillover of demand into the unregulated sector. Aggregate credit limits, if any, must be translated into limits for individual borrowers, and since 'crop loans' are intended to finance cultivation, with the crop as security, the regulations require that a household's credit limit be determined by a rule based on its acreage and cropping mix. In practice, however, land that is leased is excluded from consideration, and in other respects officials seem to enjoy some latitude in interpreting the provisions of the regulations, a possibility that is allowed for in the model. Third, we impose a sequential structure, in which households first approach institutional sources and then fill unmet demand by private borrowing. This sequence reflects the fact that co-operatives, which were virtually the sole source of institutional loans, offered cheaper credit on average, at least with respect to the base interest rate—11% per annum as opposed to an average of 18.7% charged by commission agents. There is also some direct evidence regarding the sequence of borrowing. Of the 65 households which borrowed from both

1 The data employed in this study are available from the authors upon request.

2 We lack direct information regarding the transaction costs associated with borrowing from the two segments of the market. Suggestive evidence, however, is provided by data from the southern state of Andhra Pradesh. There, the difference in interest rates was far from completely eroded by higher transaction costs in the institutional sector. For cash loans, the base annual interest rate of private loans was 7.3 percentage points higher than that for institutional loans. When transaction costs are added, the gap fell to 3.8 percentage points (Chun, 1993).
segments, 40 spaced their deals more than a month apart; and of these, 28 borrowed first from co-operatives. The probability of observing 28 successes in 40 Bernoulli trials when the probability of success is 50% is 0.008, well under 5%. Over the whole sample, borrowing from co-operatives reached its peak in November, while borrowing from unregulated sources did so a month later. Fourth, we incorporate the possibility for private loans to be interlinked with crop marketing.

We have three specific goals in view: first, to assess the strength and direction of various influences both on the household's demand for finance and on the amount and associated terms that institutions and private lenders are prepared to offer it; second, to establish the extent of rationing in the regulated market and the associated spillover of demand into the unregulated market; and third, to analyze the household's choice of whether to enter into an interlinked contract, and the extent to which such a condition enlarges the amount of finance offered. Thus, we attempt not only to develop and estimate a model of a particular sort of parallel market structure, but also to integrate it fully with a model of interlinking (recent papers include Bell, 1990; Conning, 1995; Floro and Ray, 1992; Hoff and Stiglitz, 1995; Jain, 1995; and Kochar, 1993).

Of the empirical studies of rationing in rural credit markets, Kochar's (1993) is the closest in approach to ours. In one important respect, her formulation is also more general: she does not impose the assumption that regulated credit is cheaper for all households, though, lacking data on transactions costs, she defines the cost of borrowing as the rate of interest alone. On the other hand, she does not allow for the effects of (potential) spillovers. Nor does she deal with interlinking, information on which was not collected in her source of data.

The plan of the paper is as follows: Section 2 sets out the model, in which farming households are confronted with non-exclusive menus of contracts offered by regulated and unregulated lenders. Section 3 specifies the associated econometric structures, and the data are described in Section 4. The results of estimation, the statistical performance of the models, the extent of rationing in the regulated segment of the market, and the determinants and effects of interlinking are discussed in Section 5. The paper concludes with a discussion of the implications for public policy.

2. A model of the credit market

Our goal in this section is to provide a theoretical framework to guide the specification of the econometric model, so we pay particular attention to the institutional features of the credit markets in rural Punjab. The emphasis here is on a clear statement of our assumptions and the results on which we will draw in subsequent sections. The treatment is intuitive and graphical, the technical details being consigned to the Appendix.

A household has endowments of land, \( H \), and labor, \( L \), which it supplies inelastically. It faces parametric prices \( (w, p, r_1) \) for labor \( (L) \), intermediate
inputs \((N)\), and institutional credit, respectively. There is no market for tenancies, so that land is a fixed factor. The household’s production possibilities are given by \(\theta F(L, N; \bar{H})\), where \(F(\cdot)\) is an increasing, strictly concave and differentiable function, and \(\theta\) is an i.i.d. random variable representing the state of nature, which is revealed only after all inputs have been committed. In addition to income from cultivation, the household has an income of \(\bar{Y}\) from other sources, where \(\bar{Y}\) is assumed to be riskless and to accrue after the time when working capital must be committed. Let the household’s endowment vector be denoted by \(\bar{Z} = (\bar{H}, \bar{L}, \bar{Y})\).

For simplicity, we assume that households do not have access to an asset that can be saved across periods, and that within each period, earnings from the net supply of labor and any loans obtained can be used only to finance expenditures on variable inputs. Realized output and income from other sources are allocated between consumption, \(Y\), and the repayment of debt at the end of the period. In the absence of a loan, therefore, the liquidity constraint on the purchase of variable inputs is

\[
wL + pN = w\bar{L}
\]  
(1)

and the household consumes

\[
Y_D \equiv \theta F(L, N; \bar{H}) + \bar{Y}
\]  
(2)

Let the household’s preferences over lotteries be represented by the von Neumann–Morgenstern utility function \(U(Y)\), and let the present discounted value of the (stationary) stream of expected utility at the household’s optimum be denoted by \(V_D\).

The purpose of working capital in this model is to relax the liquidity constraint. If the household borrows an amount \(Q\), then variable inputs are chosen to satisfy\(^3\)

\[
wL + pN = w\bar{L} + Q \equiv K
\]  
(3)

With the household taking \(w\) and \(p\) as given, we can replace \(F\) in (2) by \(G(K; \bar{H}) \equiv \text{Max}_{L,N} F\) subject to (3). Given \(K\), the optimal \(L\) and \(N\) are functions of \(K, w, p,\) and \(\bar{H}\).

As output is risky, the question arises: what happens when realized income at the end of the season does not cover debt obligations? In the present setting, the only collateral demanded by lenders, regulated and unregulated alike, is the crop itself. If the lender is a commission agent, we make the reasonable assumption that he can, when necessary, seize the entire crop, for the output will be sold either through him or through another member of the same trading community. If the lender is a co-operative, however, this assumption appears somewhat heroic, even if there were no private lender involved to exercise debt

\[^3\text{We assume that the amount borrowed has to be spent on purchases of variable inputs. In other words, it cannot be consumed or saved for future purchases of inputs. Admittedly, this is a strong assumption. With it, (3) will always be binding.}\]
seniority. All lenders are nevertheless in a position to deny a household access to future loans if it defaults, and this is exactly what co-operatives do.

For its part, the household has the option of repaying the loan at the end of the season, thereby remaining eligible for a new loan in the next period. In a stationary equilibrium, this involves selecting in advance a level of income such that any realized value below that level will trigger default. Equivalently, the household's choice variable is a trigger level of the state of nature, which is denoted by \( \theta_1 \geq 0 \). In any period, consumption is thus given by

\[
Y = Y_D \text{ if } \theta \leq \theta_1; \quad Y = Y_D - (1 + r_1)Q_1 \text{ otherwise} \tag{4}
\]

The probability of default is

\[
\Delta_1 \equiv \Pr (\theta \leq \theta_1) \tag{5}
\]

and the value of the life-time expected utility yielded by the choices\(^4\) \((Q_1, \theta_1)\) is

\[
V_1 = EU(Y) + \Delta_1 \delta V_D + (1 - \Delta_1) \delta V_1, \text{ or}
\]

\[
V_1 = [EU(Y) + \Delta_1 \delta V_D]/[1 - (1 - \Delta_1) \delta] \tag{6}
\]

where \( \delta \) is the household's discount factor and \( E \) is the expectations operator.

The household's problem is to choose \((Q_1, \theta_1)\) so as to maximize \( V_1 \) subject to (3)-(5). It is shown in the Appendix that \( V_1 \) is concave in \( Q_1 \) for any given \( \theta_1 \) and, indeed, monotonically increasing in \( Q_1 \) if the household chooses to default always. This implies that unless there is a ceiling on \( Q_1 \), the household would seek to borrow an indefinitely large amount of credit with the intention of defaulting. In reality, co-operatives impose credit ceilings and we will assume this to be the case. In the face of such a ceiling, it is readily shown that the household will not always default, even if it borrows up to this limit (see Appendix). The household will compare the life-time expected utility from borrowing the ceiling amount and choosing the associated optimal value of \( \theta_1 \) with that yielded by an interior maximum of \( V_1 \) with respect to \( Q_1 \) and \( \theta_1 \), if such a maximum exists. In the latter case, let \((Q_0^1, \theta_1^0)\) denote the associated choice vector, so that the notional demand for institutional credit at the parametric prices \((w, p, r_1)\) is

\[
D^0_1 = D^0_1(w, p, r_1; \tilde{Z}) \equiv Q_1^0
\]

By definition, the iso-\( V_1 \) contour passing through \((D^0_1, r_1)\) in \((Q, r)\)-space attains a maximum there (see Fig. 1).

We turn next to the private lender, who can seize the entire crop in addition to denying the household loans in the future if the crop is insufficient to pay off the current loan in full. The household cannot default in any state of nature of its choosing, as in the case of a loan from the co-operative, and it will certainly

\(^4\)The continuation value, after the decision to default or not, of life-time expected utility is clearly \( V_D \) if the decision is to default since no further loans will be forthcoming. It is \( V_1 \) itself if the decision is to repay and an optimal default policy is pursued thereafter, since the environment is stationary. Hence the expected continuation value is \( \Delta_1 V_D + (1 - \Delta_1) V_1 \).
Fig. 1. The borrower's opportunity set and optimum
choose to repay if the value of the crop exceeds what it owes. If it has borrowed $Q_2$ at the rate $r_2$, its consumption in this period will be

\[ Y = \bar{Y} \quad \text{if} \quad \theta G(.) \leq (1 + r_2)Q_2; \quad \text{and} \quad Y = Y_D - (1 + r_2)Q_2 \quad \text{otherwise} \quad (4') \]

The probability of a default is

\[ \Delta_2 \equiv \Pr (\theta < \theta_2) \quad (5') \]

where $\theta_2 \equiv [(1 + r_2)Q_2]/G(w\bar{L} + Q_2; \bar{H})$. It is clear from a comparison of (4) and (5) with (4') and (5') that even if $(Q_1, r_1) = (Q_2, r_2)$, the household will attain different levels of life-time expected utility under the two contracts; for their provisions concerning collateral are different. It is proved in the Appendix that the maps of iso-$V_1$ and iso-$V_2$ contours in $(Q, r)$-space are not the same, and hence that the household’s optimal choice of credit at the same parametric rate will differ, that is, $D^1_1(r; \cdot) \neq D^2_1(r; \cdot)$.

A further consequence of this difference in contractual provisions is that when the household obtains loans from both sources, with the private lender enjoying debt seniority, (4) and (5) must be correspondingly modified. There are three possible cases: both loans are repaid; the institutional loan is defaulted; and both loans are defaulted. Let

\[ Y_{12} \equiv \theta G(w\bar{L} + Q_1 + Q_2; \bar{H}) - (1 + r_2)(Q_1 + Q_2) + (r_2 - r_1)Q_1 + \bar{Y} \quad (7) \]

The household’s consumption in the three cases is

\[ Y = \bar{Y} \quad \text{if} \quad \theta_2 > \theta; \quad Y = Y_{12} + (1 + r_1)Q_1 \quad \text{if} \quad \theta_1 > \theta \geq \theta_2; \quad Y = Y_{12} \quad \text{otherwise} \quad (4'') \]

In general, the associated map of life-time expected utility contours in $(Q, r)$-space will be different from either of the ‘pure’ cases dealt with above. We return to this point below, after discussing the supply side of the market.

Institutional lenders must offer loans at the regulated rate. How much an institution offers to a particular household, however, is usually a matter of discretion. Short of loanable funds and mindful of asymmetries of information, it will impose a credit ceiling (or ration) on applicants for loans. The ration reflects not only the need to preclude indefinitely large demand for credit by any household, but also the amount of loanable funds with the lender. If a household faces a ration $R_1$, its opportunity set in this segment of the market will be

\[ S_1 = \{(Q_1, r) : 0 \leq Q_1 \leq R_1, r = r_1\} \quad (8) \]

The two cases where a transaction occurs are depicted in the three panels of Fig. 1. In the first panel, the household realizes its notional demand for credit at the regulated rate; in the others, it is rationed in the institutional segment of the market.

The supply side of the unregulated segment of the market is viewed as a system of contestable monopolies, in which each borrower has a contract with one private lender. In the larger marketing centers there are usually two or three hundred active commission agents. Although they serve clients drawn
from fairly well-defined territories, it is plausible that the total number of agents is sufficiently large to ensure a strong measure of effective competition among them. It is assumed that these lenders are risk-neutral, that their opportunity cost of funds (inclusive of the cost of managing loans to households) is constant at $r_0$, and that $r_0 > r_1$. A private lender has debt seniority over the co-operative, so his expected profit from a loan of $Q_2$ at the rate $r_2$ is

$$E\pi = \int_0^{\theta_2} \left[ \theta G(K; \bar{H}) - (1 + r_2)Q_2 \right] h(\theta) \, d\theta + (r_2 - r_0)Q_2$$

(9)

where the arguments of $F(\cdot)$ are chosen by the borrower. In general, therefore, $E\pi$ will depend not only on the terms of the unregulated loan, but also on $\bar{Z}$ and the terms of the regulated loan $(Q_1, r_1)$, if there is one. Which of the components of $\{\bar{Z}, Q_1\}$ the private lender can actually observe is another matter, to which we return in a later section.

Under the above assumptions, free entry ensures that the expected profit from each and every contract is zero. Denote the zero-expected profit contour in $(Q_2, r_2)$-space by

$$r_2 = g(Q_2; x)$$

(10)

where $x$ denotes those of the borrower's characteristics that the lender can observe and are relevant to contractual performance. The contour in question is the complete menu of private contracts offered to a potential borrower with observable characteristics $x$.

Since the integrand in (9) is negative in the interval $[0, \theta_2]$, it follows at once that $r_2 > r_0$ for all $Q_2 > 0$. In the limit, as $Q_2$ becomes very small, $\theta_2$ does likewise; so that a lender will just break even on a very small loan at a rate just above $r_0$. Hence $(0, r_0)$ is a limit point satisfying (10). That $g(\cdot)$ is an increasing function of $Q_2$ is established in the Appendix.

Combining (8) and (10) yields the boundary of the borrower's opportunity set in the space of $Q$ ($\equiv Q_1 + Q_2$), the total amount borrowed in both segments of the market, and $r$, the rate of interest. This is made up of the reverse L-shaped offer schedule from the institution and the zero-expected profit contour of a private lender. In Fig. 1, $g(Q_2; x)$ is drawn with origin $(R_0, 0)$, to reflect the fact that the borrower seeks institutional finance first.

Given that the household has the opportunity to borrow in the unregulated segment of the market at this stage, we return to its demand for credit. Recalling (4'') and (7) above, three observations are in order. First, if $R_1 > 0$,
then in the region bounded to the left by the vertical line through \((R_1, 0)\) and below by the menu \(g(Q_2; x)\), the associated map of life-time expected utility contours will be different from either of the two pure cases analyzed above. Second, if the optimum involves \(Q_2^0 > 0\), then the fact that \(g(Q_2; x)\) is upward-sloping implies that \(Q_2^0\) is less than the borrower’s notional demand for private credit at the rate \(r_2 = g(Q_2^0; x)\). Such an outcome is depicted in panel 3 of Fig. 1. This is the spillover case. Third, when the household obtains \(Q_1\) at the first stage and borrows at the second stage, it receives a rent which effectively augments \(\bar{Y}\) by the amount \((r_2 - r_1)Q_1\). This affects not only \(\bar{Y}\) (for any given \(\theta\) and choice of \(L\) and \(N\)), but also the values of \(\theta\) at which default occurs. Thus, the map of the borrower’s indifference contours in this region of \((Q, r)\)-space is altered by the presence of the rent, even if the borrower is risk-neutral.

To sum up the implications of the findings thus far for the econometric specification of the demand side of the model: first, the demand functions for institutional and informal credit will have different parameter values for the same regressors; second, the (predicted) amount of institutional credit should enter as a regressor in the demand function for informal credit; and third, since the rent \((r_2 - r_1)Q_1\) augments \(\bar{Y}\), it follows that if absolute risk-aversion is decreasing with income, then a unit increase in \(Q_1\) will induce a decrease in the notional demand for informal credit of less than one unit (Srinivasan, 1972). Thus, we have a specific testable hypothesis concerning the coefficient of \(Q_1\) in the demand function for informal credit.

To conclude this section, we address interlinking with marketing, because the main private lenders are commission agents. An interlinked contract of this sort reduces the borrower’s marketing options and commits him to pay a commission fee on sales, while securing business for the lender in advance of the peak demands of the harvest season when his time is particularly valuable. Thus, the contract alters both the borrower’s indifference map and the position of the zero-expected profit contour in \((Q_2, r_2, t)\)-space, where \(Q_2 \geq 0, r_2 \geq 0\), and the binary variable \(t\) equals one in the presence of a tying arrangement and is zero otherwise. For any given rate of interest, a tying provision will normally increase the amount the lender is prepared to advance. In Fig. 2, \(g(Q_2, t = 1)\) is therefore drawn to the right of \(g(Q_2, t = 0)\). The choice between a tied and an untied transaction, however, rests with the borrower, by virtue of the assumption that there is competition among lenders. In Fig. 2, the solutions under each arrangement are depicted as \(C\) and \(T\), respectively, with \(C\) or \(T\) being chosen according as \(V(Q_2^0, r_2^0, t = 0) \geq V(Q_2^1, r_2^1, t = 1)\). The straight line \(f(\cdot)\), which connects \(C\) and \(T\), will play an important role in the formulation of the econometric model that follows.

### 3. The econometric model

Figure 3 contains a summary of the timing of decisions in the model of Section 2. The model exhibits the following key features, which we incorporate into the econometric structure.
RATIONING, SPILLOVER, AND INTERLINKING

Endowment realized, regulated interest rate and other prices set
Cooperatives determine ration based on observed characteristics of household
Notional demand of household determined, based on endowment, interest rate and prices
Transact minimum of demand and ration
Private lender offers all contracts \( r_2, Q_2, r \in \{0, 1\} \) which yield zero expected profit, depending on observed characteristics
Household chooses contract which maximizes expected utility, based on endowment, prices, and quantity and interest rate transacted in regulated market

Regulated Segment

Unregulated Segment

(i) Borrowing is sequential. Each household first calculates its notional demand for institutional credit at the parametric interest rate \( r_1 \) and transacts the minimum of this demand and the ration allocated to it by the co-operative. A switching regression model, therefore, is appropriate to estimate household
demand for credit, and the supply of credit offered to that household by the co-operative.

(ii) Households then have the option of approaching private lenders and selecting a contract from the menus offered to them.

(iii) The expected profit yielded by a credit contract depends upon any characteristic of the borrower which affects input use. Any such characteristic which is observed by the lender, therefore, also influences the menu of contracts offered by him. This raises serious issues of identification.

(iv) The demand for unregulated credit depends upon the amount that the household has borrowed from the co-operative.

(v) The borrower must also choose between a tied and an untied contract. This choice depends on the size of the associated shift in the lender’s menu of offers, and on the quantity of the output that the borrower expects to produce.

3.1. The regulated market

In the institutional segment of the market, the short side of the market is observed and there is unknown sample separation. Both the household’s demand for credit and the ration that the co-operative is prepared to offer have a positive probability of being zero, so the model incorporates the possibility that the observed amount transacted can be censored at zero. Let the notional demand for credit be given by

\[ D_1 = \max (0, y_1 \beta_1 + \lambda_1 r_1 + v_1) \]  

and the ration by

\[ R_1 = \max (0, x_1 \alpha_1 + u_1) \]  

\( x_1 \) and \( y_1 \) are vectors of exogenous household characteristics which respectively affect the household’s demand for credit and the ration which the co-operative is prepared to offer the household. As \( y_1 \) is the same for all borrowers, \( \lambda_1 \) cannot be estimated; but this term will be retained for expositional purposes below. Then the observed amount of regulated credit transacted, \( Q_1 \), is

\[ Q_1 = \min (D_1, R_1) \]  

Estimates of \( \alpha_1 \) and \( \beta_1 \) are obtained by maximizing the likelihood function implied by (11)–(13), which comprises two observationally distinguishable cases, each comprising two indistinguishable sub-cases.

Case 0. \( Q_1 = 0 \) (67 observations). The contribution to the likelihood of these observations is the probability that \( D_1 \leq 0 \), or \( D_1 > 0 \) and \( R_1 \leq 0 \): that is

\[ v_1 \leq -y_1 \beta_1 - \lambda_1 r_1, \text{ or} \]

\[ v_1 > -y_1 \beta_1 - \lambda_1 r_1 \text{ and } u_1 \leq -x_1 \alpha_1 \]  

\[ (14) \]
Case 1. \(Q_1 > 0\) (129 observations). The contribution to the likelihood of these observations is the probability that \(D_1 = Q_1\) and \(Q_1 < R_1\), or \(R_1 = Q_1\) and \(Q_1 < D_1\) that is

\[
\begin{align*}
\nu_i &= Q_1 - y_1 \beta_1 - \lambda_1 r_1 \text{ and } u_i > Q_1 - x_1 \alpha_1, \text{ or} \\
nu_i &= Q_1 - x_1 \alpha_1 \text{ and } u_i > Q_1 - y_1 \beta_1 - \lambda_1 r_1
\end{align*}
\]

(15)

The disturbance terms \((u_i, \nu_i)\) are assumed to be i.i.d. as a bivariate normal with parameters \(\sigma_u, \sigma_\nu\) and \(\rho_{uv}\). This is not quite the standard model of market disequilibrium (see, for example, Maddala, 1983) in that there is a mass point of observations at \(Q_1 = 0\).⁶

3.2. The unregulated market

The model of transactions in the private segment of the market is less familiar. First, consider the supply side of the market. To make estimation tractable, we assume that the zero-expected profit contour \(g^\sim(\cdot)\) is linear: 

\[
\begin{align*}
g^\sim(r; x) &= x_2 \alpha_2 + t x_2^2 \alpha_2^t + \gamma_2 r_2 + u_2, \text{ where } x_2 \text{ is a subset of the variables included in } x_2.
\end{align*}
\]

As the sample size is rather small, we include only the constant term in \(x_2^t\), so that a tied contract \((t = 1)\) causes a parallel shift of the zero-expected profit contour by a fixed amount. Then the two zero expected profit contours (one for \(t = 0\), the other with \(t = 1\)) are defined by those triplets \((Q_1, r_2, t)\) [where \(Q_1 \geq 0, r_2 \geq 0, t \{0, 1\}\] satisfying

\[
Q_2 = x_2 \alpha_2 + t x_2^2 \alpha_2^t + \gamma_2 r_2 + u_2
\]

(16)

Now turn to demand. Our goal is to estimate the functions \(f(r_2; t; y_2)\) and \(D(r_2, t; y_2)\) in Fig. 2. \(D(\cdot)\) is the amount that the household would borrow at a parametric rate of interest \(r_2\). \(f(\cdot)\) plays a role analogous to \(D(\cdot)\) in this principal-agent framework. As defined in Section 2, \(f(\cdot)\) is the line which joins the two points \((Q_2^1, r_2^1, t = 1)\) and \((Q_2^0, r_2^0, t = 0)\), which are the borrower's optimal choices of contract subject to the lender's zero-expected profit constraint with and without a tie respectively. We assume \(D(\cdot)\) is linear and that \(f(\cdot)\) is parallel to \(D(\cdot)\).

Given the previous assumption regarding the shape of \(g(\cdot)\), the assumption that \(f(\cdot)\) and \(D(\cdot)\) are parallel is equivalent to an assumption on the underlying preference map. To be precise, the assumption that \(f(\cdot)\) and \(D(\cdot)\) are linear and parallel is equivalent to the assumption that the value function 

\[
V(r_2, Q_2, t; Q_1, r_1)
\]

has the property \((\partial V/\partial Q_2)/(\partial V/\partial r_2) = \psi(Q_2 + \lambda_2 r_2)\), where \(\lambda_2\) is a negative constant and \(\psi(\cdot)\) is continuous and differentiable. Hence, all loci along which the marginal rate of substitution in the space of \((Q_2, r_2)\) is constant are parallel lines with slope \(1/\lambda_2\). This holds, in particular,

⁶The formulae and Gauss code which provide the values of the likelihood function and its derivatives for each observation for any choice of parameters are available from the authors upon request.
for \( MRS = 0 \), the locus of which is the notional demand schedule for private credit. The function \( f(\cdot) \), therefore, is defined as those pairs \((Q_d, r_2)\) satisfying

\[
Q_d = y_2 \beta_2 + \lambda_2 r_2 + v_2
\]

where it will be recalled from section 2 that \( y_2 \) includes \( Q_1 \). Since \( D_2(\cdot) \) is parallel to \( f(\cdot) \), we have

\[
D_2 = y_2 \beta_2^2 + \lambda_2 r_2 + v_2
\]

The horizontal distance between the lines \( f(\cdot) \) and \( D_2(\cdot) \) depends, in general, upon the borrower's characteristics. Hence we specify the relation \( \beta_2^2 = \beta_2 + \kappa \), where \( \kappa \) is a parameter vector some of whose elements we restrict to zero.

Equations (17) and (18) specify the demand side of the model.

To complete the model of the unregulated segment of the market, we recall that the decision of whether to accept a tied deal lies in the hands of the borrower and is endogenous. Since, from (16), a tie yields extra finance in the amount \( x_2 \alpha_2 \), which is a benefit to the borrower, we define the unobservable index function

\[
t^* = z \delta_0 + \delta_1 (x_2 \alpha_2') + \eta
\]

such that we observe \( t = 1 \) if \( t^* > 0 \), and \( t = 0 \) otherwise. We hypothesize that the costs of tying increase with the prospective size of the crop to be marketed, so that \( z \) should comprise those variables that affect the marketed surplus.

The model of the private credit market is estimated by maximizing the likelihood function implied by (16)—(19), along with the non-negativity constraints on \( Q_2 \) and \( r_2 \). The likelihood function comprises five observationally distinguishable cases. Cases 1 and 2 arise when \( Q_2 = Q_d \) at \( r_2 > 0 \), with and without a tie, respectively, which are depicted in Fig. 2 as points \( T \) and \( C \).

**Case 1.** \( Q_2 > 0, r_2 > 0, t = 1 \) (48 observations). The contribution to the likelihood of these observations is the probability that

\[
v_2 = Q_2 - [y_2 \beta_2 + \lambda_2 r_2] \quad \text{and} \quad u_2 = Q_2 - x_2 \alpha_2 - x_2 \alpha_2' - \gamma_2 r_2 \quad \text{and} \quad \eta > -z \delta_0 - \delta_1 (x_2 \alpha_2')
\]

**Case 2.** \( Q_2 > 0, r_2 > 0, t = 0 \) (eight observations). This case is observed when

\[
v_2 = Q_2 - [y_2 \beta_2 + \lambda_2 r_2] \quad \text{and} \quad u_2 = Q_2 - x_2 \alpha_2 - \gamma_2 r_2 \quad \text{and} \quad \eta \leq -z \delta_0 - \delta_1 (x_2 \alpha_2')
\]

Cases 3 and 4 arise when \( r_2 = 0 \), but positive quantities of credit are transacted.
Case 3. \(Q_2 > 0, \ r_2 > 0, \ t = 1\) (22 observations). The contribution to the likelihood function is the sum of the probabilities of the two following observationally indistinguishable sub-cases: either \(Q_2 = D(r_2 = 0)\) and \(Q_2(r_2 = 0, t = 0) < Q_2 < Q_2(r_2 = 0, t = 1)\); that is
\[

v_2 = Q_2 - y_2^2 \beta_2^d, \quad \text{and}
\]
\[
Q_2 - x_2 \alpha_2, > u_2 > Q_2 - x_2 \alpha_2 - x_2^1 \alpha_2, \quad \text{and}
\eta > -z \delta_0 - \delta_1(x_2^1 \alpha_2^2) \quad (22a)
\]
or \(Q_2 = Q_2(r_2 = 0, t = 1)\) and \(Q_2(r_2 = 0) < Q_2 < D(r_2 = 0)\); that is
\[
Q_2 - y_2 \beta_2 > v_2 > Q_2 - y_2 \beta_2, \quad \text{and}
\]
\[
u_2 = Q_2 - x_2 \alpha_2, \quad \text{and}
\eta > -z \delta_0 - \delta_1(x_2^1 \alpha_2^2) \quad (22b)
\]

Case 4. \(Q_2 > 0, \ r_2 > 0, \ t = 0\) (four observations). There are two observationally indistinguishable sub-cases: either \(Q_2 = D(r_2 = 0)\) and \(Q_2(r_2 = 0, t = 0) > Q_2\); that is
\[

v_2 = Q_2 - y_2^2 \beta_2^d, \quad \text{and}
\]
\[
u_2 > Q_2 - x_2 \alpha_2, \quad \text{and}
\eta < -z \delta_0 - \delta_1(x_2^1 \alpha_2^2) \quad (23a)
\]
or \(Q_2 = Q_2(r_2 = 0, t = 0)\) and \(Q_2(r_2 = 0) < Q_2 < D(r_2 = 0)\); that is
\[
Q_2 - y_2 \beta_2 > v_2 > Q_2 - y_2 \beta_2, \quad \text{and}
\]
\[
u_2 = Q_2 - x_2 \alpha_2, \quad \text{and}
\eta < -z \delta_0 - \delta_1(x_2^1 \alpha_2^2) \quad (23b)
\]

Finally, Case 5 arises when no unregulated credit is transacted.

Case 5. \(Q_2 = 0\) (114 observations). There are two observationally indistinguishable sub-cases: either \(D(r_2 = 0) \leq 0\); that is
\[

v_2 \leq -y_2^2 \beta_2^d \quad (24a)
\]
or \(D(r_2 = 0) > 0\) and \(D(r_2') < 0\), where \(r_2'\) satisfies \(Q_2(r_2') = 0\); that is
\[

v_2 > -y_2 \beta_2^d, \quad \text{and}
\]
\[
\eta < -y_2 \beta_2^d + \frac{\lambda_2}{\gamma_2} \quad (24b)
\]

where \(\epsilon = v_2 - (\lambda_2 / \gamma_2) u_2\), \(u_2,\ v_2\), and \(\eta\) are assumed to be distributed as a trivariate normal, i.i.d. across observations.

We close this section with a remark on identification. As noted in Section 2, those of the borrower's characteristics which the lender can observe and which also affect contractual performance will influence the menu of contracts offered by the lender. Since these characteristics will also, in general, affect the shape
and position of the borrower's indifference map, it is hard to make a convincing case that any variable that appears in \( x_2 \) should not also be included in \( y_2 \). Thus, exclusion restrictions cannot be used to identify the parameters of the \( g(\cdot) \) and \( f(\cdot) \) functions. Identification, therefore, must rest instead on the non-linearity of the model.\(^7\) While this is unsatisfactory from an econometric point of view, it is an unavoidable consequence of a central result of our theoretical model.

4. The survey and the data

The survey was canvassed between September 1980 and May 1981 as part of the World Bank's research project RPO-671-89. It covered the rabi (winter) season, in which the main crop is wheat. The sampling proceeded in three stages. At the first stage, the districts of Jullunder and Ferozepur were selected purposively. Both districts belong to the central Punjab plain, which is very extensively irrigated and commercialized.

At the second stage, five villages were chosen from each district, again purposively, in order to capture variations in public infrastructure. A census of all households in each village was carried out at the start of the survey period. The information collected covered each household's demographic and social characteristics, its primary and secondary occupations, its endowments of land, livestock, and machinery, and the amounts of land it was leasing in and out.

At the third stage, a total of 40 households were drawn from each village over two rounds. The census was used to assign all households to one of ten socio-economic categories. One landlord, six owner cultivators, six owner-tenants, and four pure tenants were then drawn, with probability proportional to size of landholding. One household was drawn, by simple random sampling, from each of the remaining categories—four types of laborers, traders, moneylenders, and others—yielding a primary sample of 24 households at the first round.\(^8\) After these households had been followed for two months, a list was drawn up of all the other households in the village with which the primary sample claimed to have had some dealings in the main markets during the reference period. At the second round, a simple random sample of 16 households was drawn from that list.\(^9\)

\(^7\) To see this most simply, suppose that all of the observations fell into case 2. Since \( x_2 \subset y_2 \) it would be impossible to distinguish \( \alpha_2 \) from the equivalent elements of \( \beta_2 \). The information provided by the observations in cases 3–5 permits the separate identification of these parameters, because they are differently affected by the non-negativity constraints of the model. Observe in particular that \( \alpha_2 \) and \( \beta_2 \) do not enter symmetrically into (24a) and (24b).

\(^8\) No registered moneylenders were found in these villages, and the traders were shopkeepers or small-scale dealers who engaged in only petty lending. The commission agents from whom cultivators borrowed were usually resident in market towns, and so fell outside the sampling frame.

\(^9\) Clearly, the probability that a household appeared as a candidate for selection at the second stage, conditional on its not being chosen at the first stage, depended upon the probability that at least one of the households with which it had had transactions was chosen in the first stage. For a detailed description of the sampling scheme and how the probabilities were estimated, see Srinivasan and Sussangkarn (1984). The summary statistics and econometric results reported in this paper reflect these sampling weights.
RATIONALING, SPILLOVER, AND INTERLINKING

Table 1
Credit transactions of sampled households (Rs)

<table>
<thead>
<tr>
<th>Institutions</th>
<th>Non-Borrowers</th>
<th>Tied</th>
<th>Untied</th>
<th>Subtotal</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-borrowers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number</td>
<td>50</td>
<td>14</td>
<td>3</td>
<td>17</td>
<td>67</td>
</tr>
<tr>
<td>amount (public)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(private)</td>
<td>0</td>
<td>3,081</td>
<td>963</td>
<td>2,622</td>
<td></td>
</tr>
<tr>
<td>Borrowers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number</td>
<td>64</td>
<td>56</td>
<td>9</td>
<td>65</td>
<td>129</td>
</tr>
<tr>
<td>amount (public)</td>
<td>3,674</td>
<td>3,236</td>
<td>5,229</td>
<td>3,437</td>
<td></td>
</tr>
<tr>
<td>(private)</td>
<td>0</td>
<td>5,139</td>
<td>3,871</td>
<td>5,279</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>114</td>
<td>70</td>
<td>12</td>
<td>196</td>
<td></td>
</tr>
</tbody>
</table>

This paper restricts attention to the 196 (out of an original sample of 400) households which were cultivating and members of a co-operative. A summary of their transactions in the two segments of the credit market is set out in Table 1. The great majority of Punjabi farmers were members of a co-operative, so this particular restriction is unlikely to introduce significant selectivity bias. The restriction to cultivating households may exclude some households which chose not to cultivate in anticipation of not having access to credit. This complication is also ignored.

Table 2 lists the data which are used in the analysis. The amount borrowed in the regulated market is simply the value of loans taken from the co-operative during the six-month period from September to February and used to finance consumption and working capital. The amount borrowed in the unregulated market is similarly defined. The unregulated market interest rate is the average (weighted by loan value) interest rate on such loans.

A dummy variable distinguishes the more developed, commercialized, and densely settled district of Jullunder from its more traditional and sparsely populated neighbor, Ferozepur. This dummy variable will capture the effects of district-level variations in prices, infrastructure, soils, and irrigation. Furthermore, while they are closely regulated in some respects, co-operatives are in some measure local institutions and may function differently across the two districts. We also include a crude measure of village-level endowments, the
### Table 2

<table>
<thead>
<tr>
<th>Description</th>
<th>Units of measurement</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>formal credit</td>
<td>Rs. 1,000</td>
<td>2.50</td>
<td>2.85</td>
<td>0.00</td>
<td>20.00</td>
</tr>
<tr>
<td>informal credit</td>
<td>Rs. 1,000</td>
<td>2.44</td>
<td>4.09</td>
<td>0.00</td>
<td>30.00</td>
</tr>
<tr>
<td>informal interest rate</td>
<td>—</td>
<td>7.42</td>
<td>10.06</td>
<td>0.00</td>
<td>30.00</td>
</tr>
<tr>
<td>village endowment ratio</td>
<td>adult males/hectare</td>
<td>0.99</td>
<td>0.39</td>
<td>0.50</td>
<td>2.18</td>
</tr>
<tr>
<td>district dummy (Jullhnder = 1)</td>
<td>—</td>
<td>0.31</td>
<td>0.46</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>dummy for high caste</td>
<td>—</td>
<td>0.52</td>
<td>0.50</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>dummy for medium caste</td>
<td>—</td>
<td>0.27</td>
<td>0.44</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>family size</td>
<td>persons</td>
<td>7.27</td>
<td>3.73</td>
<td>1.00</td>
<td>25.00</td>
</tr>
<tr>
<td>dependency ratio</td>
<td>—</td>
<td>40.94</td>
<td>19.21</td>
<td>0.00</td>
<td>77.78</td>
</tr>
<tr>
<td>working males in agriculture</td>
<td>as a proportion of family size</td>
<td>—</td>
<td>29.67</td>
<td>13.59</td>
<td>0.00</td>
</tr>
<tr>
<td>liquid assets</td>
<td>Rs. 1,000</td>
<td>2.14</td>
<td>4.64</td>
<td>0.00</td>
<td>50.16</td>
</tr>
<tr>
<td>value of tractors and draft animals</td>
<td>Rs. 1,000</td>
<td>11.11</td>
<td>18.68</td>
<td>0.00</td>
<td>83.00</td>
</tr>
<tr>
<td>land owned</td>
<td>Rs. 10,000</td>
<td>24.53</td>
<td>25.80</td>
<td>0.00</td>
<td>206.00</td>
</tr>
<tr>
<td>visible assets</td>
<td>Rs. 10,000</td>
<td>37.70</td>
<td>53.25</td>
<td>0.00</td>
<td>241.00</td>
</tr>
<tr>
<td>predicted value of formal credit</td>
<td>Rs. 1,000</td>
<td>2.80</td>
<td>3.57</td>
<td>0.00</td>
<td>21.74</td>
</tr>
</tbody>
</table>

*The average interest rate reported in Table 2 differs from that reported in the Introduction because non-borrowers (with interest rates set at zero) are included in the calculation of the mean interest rate in the table.

The number of adult males per hectare of operational land holdings, which may affect cultivation decisions.

Family size, the dependency ratio, and the proportion of full-time male workers engaged in agriculture are the main demographic variables. The last would reduce the demand for credit in a riskless world with perfect markets, since each such worker can earn wages to finance outlays on other inputs, or work on the family farm to reduce outlays on hired hands. In a risky world lacking a complete set of insurance markets, all three affect the risk-averse household’s choice of input levels and therefore the demand for loans.

The household’s caste has no place in the co-operative’s regulations; but in villages torn by caste conflicts, it could still play an important role in determining how big a loan the household is granted. In addition to the possibility that caste restricts the household’s activities, caste might also be an important determinant of a household’s social network and access to information, and therefore affect the demand for credit and the supply of unregulated loans.

The household’s assets are disaggregated as follows. First, the more draft power a farmer owns, the smaller will be his demand for hired services, though his demand for animal feed and/or diesel fuel will be higher. Second, the farmer’s liquid assets are measured as the sum of household stocks of intermediate inputs, cash, and short-term bank deposits. Ample endowments of these liquid assets should reduce the household’s demand for credit. Third, land is the most important form of wealth. In order to account for variations in land quality as well as the extent of landholdings, we use the value of land owned.
At the close of Section 3, we emphasized that all of the would-be borrower’s characteristics that the lender can observe should, in principle, influence the terms of the contract he offers to the borrower in question. In the case of the co-operatives, however, the regulations define a very narrow basis for the determination of what we have called the household’s ration. So-called crop loans are supposed to be based on the household’s acreage and cropping mix. In practice, matters are handled rather differently. Co-operatives take note of the borrower’s readily visible assets, such as land and draft power, when deciding how large a loan to advance. Such assets arguably indicate the probability that enough output will be produced for the farmer to repay the loan without difficulty. The value of visible assets does not include the value of what we have called liquid assets, the value of which is not directly observable by any lender.

This last exclusion from the ration eq. (12) is not arbitrary and, in principle, it ensures that the demand equation of the model defined by eqs (11)–(13) is identified. We have gone further, however, by excluding the three demographic variables. These could be ascertained by an exceptionally diligent and curious official of a co-operative, and would be known to a fair number of fellow villagers. We justify their exclusion on the grounds that the co-operatives function rather bureaucratically, and that whereas collateral is a central word in banking language and regulations in Punjab, terms like family size and composition are not.

5. Estimation and predictive performance

In this section we report the parameter estimates of the models specified in (14)–(15) and (20)–(24). We use these estimates to assess the extent of rationing in the regulated market, the degree of spillover of unmet demand into the unregulated market, and the quantitative impact of an agreement to tie the loan to a marketing deal on the terms of an unregulated loan.

5.1. Regulated market estimates

The parameter estimates and asymptotic t-ratios are set out in Table 3. Most of the parameters have the expected signs and are significant at the 5% level or better.

Beginning with notional demand, to the extent that a higher village man–land ratio is associated with a lower wage rate, it will be associated also with a lower demand for production credit if the demand for all variable inputs is fairly inelastic with respect to the wage rate. That notional demand should be higher in Jullunder, all else being equal, is not surprising, this being the more advanced of the two districts.

With low- and scheduled caste households as the reference group, the dummies for both medium and high caste are positive and highly significant. These results may reflect the fact that women from higher caste households rarely work in the fields, thereby creating more demand for both hired hands
and the working capital to pay for them. If relative risk aversion increases with 
*per capita* wealth, a larger family will increase the household's notional demand 
for credit by shifting its portfolio in favor of the risky activity of cultivation. In 
that case, the positive coefficient on family size is as expected. With a given 
family size, a rise in the dependency ratio implies fewer members of working 
age. In this case, the resulting effect on the demand for credit is unclear, 
because a fall in family labor supplies may be more than offset by a shift 
away from the risky activity. Conversely, a rise in the proportion of adult 
males working in agriculture implies a reduction in the number of adults 
who participate in non-farm activities. In any event, the coefficient of the 
dependency ratio is negative and highly significant, whereas that of the propor-
tion of adult males working in agriculture is positive but insignificant at con-
ventional levels.

Increased ownership of draft power reduces the household's demand for 
credit, and the coefficient is highly significant. That the coefficients of liquid 
assets and the value of land holdings are significantly positive and negative, 
respectively, is both puzzling and rather unsatisfactory. One possibility is that 
households with much land wealth lease out substantial parts of their holdings 
to tenants who must provide their own finance. Unfortunately, one cannot 
simply include leased land as a component of *y*₁, as the leasing decision is

### Table 3

**Formal market: parameter estimates and asymptotic t-ratios**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notional demand</th>
<th>Ration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>estimate</td>
<td>t-ratio</td>
</tr>
<tr>
<td>constant</td>
<td>1.049</td>
<td>1.25</td>
</tr>
<tr>
<td>village endowment</td>
<td>-2.492</td>
<td>-2.95</td>
</tr>
<tr>
<td>district</td>
<td>4.488</td>
<td>5.52</td>
</tr>
<tr>
<td>high caste</td>
<td>1.839</td>
<td>4.41</td>
</tr>
<tr>
<td>medium caste</td>
<td>6.064</td>
<td>7.59</td>
</tr>
<tr>
<td>visible assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>family size</td>
<td>1.397</td>
<td>7.98</td>
</tr>
<tr>
<td>dependency ratio</td>
<td>-0.082</td>
<td>-7.95</td>
</tr>
<tr>
<td>working males in agriculture</td>
<td>0.006</td>
<td>0.33</td>
</tr>
<tr>
<td>liquid assets</td>
<td>0.013</td>
<td>5.44</td>
</tr>
<tr>
<td>value of draft power</td>
<td>-0.079</td>
<td>-5.56</td>
</tr>
<tr>
<td>land value</td>
<td>-0.085</td>
<td>-9.69</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>0.751</td>
<td>4.68</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.308</td>
<td>-0.88</td>
</tr>
</tbody>
</table>

\[ n = 196 \quad \text{log} \mathcal{L} = 336.30 \]

Correlation between predicted and actual quantity transacted: 0.60

Likelihood ratio test: The null hypothesis is that all coefficients except the constant and variance terms are zero: \( \chi^2(18) = 189.92; \ p = 0.00. \)
simultaneous with the credit transaction. Any attempt to estimate a model of the leasing decision jointly with the credit transactions considered here would run into formidable problems, however, not the least of which is the fact the household's desired transactions in credit and tenancies are influenced by the same variables. We leave this problem, therefore, for future examination. A second, not necessarily mutually exclusive, possibility is that the desired level of input-intensity falls rapidly with land wealth after some point; but an attempt to introduce a quadratic term in the value of land was unsuccessful.

Turning to the ration equation, we find that co-operatives in Jullunder offer smaller loans than their counterparts in Ferozepur. In both districts, however, co-operatives in more densely-populated villages offer larger loans. The caste dummies are both negative, but neither is significant at conventional levels. We find no evidence, therefore, of discrimination against low- and scheduled caste households in the allocation of co-operative credit. As expected, visible assets increase the level of the ration, and the estimated coefficient is highly significant.

Where the overall performance of the model is concerned, the likelihood-ratio test of the null hypothesis that all coefficients except the constants and variances are zero (Dhrymes, 1986) yields a $\chi^2 (18)$ test statistic with a value of 190 ($p = 0$). Another measure of performance is how successfully the model predicts which households borrow. Assigning households to observable regimes in proportion to their probabilities of belonging to each regime, the model has an overall success rate in assigning households correctly of 72%, compared to 58% for the naive model that assigns households randomly with probabilities equal to the actual proportion of households appearing in each regime.

5.2. The extent of rationing

We now consider the important question of which households obtained less co-operative credit than they wanted. (The subscript 1 may be dropped without ambiguity.) We take the four sub-cases in Section 3.1 in turn. First, the probability that household $j$ does not wish to borrow is

$$Pr(D_j < 0) = Pr(v_j < -y_j\beta) = \Phi(-y_j\beta/\sigma_v)$$

where $\Phi(\cdot)$ denotes the cumulative univariate normal distribution function. Second, the probability that household $j$ wishes to borrow but cannot obtain a loan is

$$Pr(D_j > 0 \text{ and } R_j < 0) = Pr(v_j > -y_j\beta \text{ and } u_j < -x_j\alpha)$$

$$= B(y_j\beta/\sigma_v, -x_j/\alpha/\sigma_u, -\rho_{uv})$$

where $B(\cdot)$ denotes the bivariate normal distribution function. Third, there are those households which borrow and realize their notional demand. In this case, we have
TABLE 4
The extent of rationing in the regulated market

<table>
<thead>
<tr>
<th>Status</th>
<th>Not borrowing</th>
<th></th>
<th>Borrowing</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not rationed</td>
<td>Rationed</td>
<td>Total</td>
<td>Not Rationed</td>
<td>Rationed</td>
</tr>
<tr>
<td>assignment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>predicted</td>
<td>11</td>
<td>54</td>
<td>65</td>
<td>23</td>
<td>108</td>
</tr>
<tr>
<td>actual</td>
<td>18</td>
<td>49</td>
<td>67</td>
<td>24</td>
<td>105</td>
</tr>
</tbody>
</table>

\[
\Pr (R_j > D_j > 0) = B(y_j\beta/\sigma_\nu, (x_j\alpha - y_j\beta)/\sigma_w, \rho_{uw})
\]

where \(\sigma^2_\nu = \sigma^2_u + \sigma^2_\epsilon - 2\rho\sigma_u\sigma_\epsilon\) and \(\rho_{uw} = \sigma_\nu(\rho\sigma_u/\sigma_\epsilon - 1)/\sigma_w\). Fourth, the probability that household \(j\) borrows but obtains only what is offered is

\[
\Pr (D_j > R_j > 0) = B(x_j\alpha/\sigma_\nu, (y_j\beta - x_j\alpha)/\sigma_\sigma, \rho_{uw})
\]

where \(\rho_{uw} = \sigma_\nu(\rho\sigma_u/\sigma_\epsilon - 1)/\sigma_w\). If households are assigned to observable regimes in proportion to the probability of belonging to each regime, then we conclude that about 80% of non-borrowers and borrowers alike were rationed in the regulated segment of the market (see the first row of Table 4).

This procedure does not, of course, take any account of errors in the assignment of households, of which there are 55 altogether. To deal with such errors, the probability that a household is rationed conditional on its being observed in a particular regime must be calculated. If a household is observed not to borrow, the probability that it is not rationed, conditional on being observed not to borrow, is the ratio of the probability that it does not want credit, i.e., \(\Pr (D_j \leq 0)\), to the probability that it is observed not to borrow, \(\Pr (D_j \leq 0, \text{or } D_j > 0 \text{ and } R_j \leq 0)\), or \(\Phi(-y_j\beta/\sigma_\epsilon)/[\Phi(-y_j\beta/\sigma_\epsilon) + B(y_j\beta/\sigma_\epsilon, -x_j\alpha/\sigma_w, -\rho)]\). The probability that it is rationed is the ratio of \(\Pr (D_j > 0 \text{ and } R_j \leq 0)\) to \(\Pr (D_j \leq 0 \text{ or } D_j > 0 \text{ and } R_j \leq 0)\).

If a household is observed to borrow, the probability that it is not rationed is the ratio of the probability that it has positive demand and is not rationed, i.e. \(\Pr (R_j > D_j > 0)\), to the probability of being observed to borrow, i.e., \(\Pr (D_j > 0 \text{ and } R_j > 0)\). The probability that the household is rationed is analogously defined. Allocating the observed counts of 67 and 129, respectively, according to these probabilities, we obtain the second row of Table 4: almost three-fourths of non-borrowing households and just over four-fifths of borrowing households were rationed. We conclude that rationing was pervasive in the regulated segment of the market.

5.3. Unregulated market estimates

It is clear that all the variables that influence the notional demand for regulated credit should also be included in (17) and (18), which define the demand side of
the model for unregulated credit. In addition, the demand for unregulated credit depends upon the (estimated) amount of credit obtained from the co-operative. For want of degrees of freedom, the number of elements of \( K \) not restricted to zero had to be kept small: we permitted the coefficients on the district and high caste dummy variables and land value to differ from zero.

Turning to supply, recall from Section 2 that an agreement to tie the credit transaction might increase the size of the loan that the lender is prepared to offer at any interest rate. We include only a constant in \( x' \) (see eq. (16)); hence \( g(\cdot) \) and \( g(\cdot; t) \) are assumed to differ only by the horizontal distance equal to the coefficient on that constant. Their regressors are otherwise identical to those in \( R_t(\cdot) \). In particular, we assume that private lenders cannot observe the size of the loan obtained from the regulated sector.

In the index function for the acceptance or refusal of a tied contract, we include predictors of the size of a farmer's crop: the larger his expected output, the less willing he would be to commit it to a particular trader.

The results of the unregulated market estimation are reported in Table 5. Before discussing them, however, two remarks are in order. First, it was impossible to obtain convergence without imposing restrictions on certain parameters: namely, that the correlation between \( \eta \) and each of \( u_2 \) and \( v_2 \) be zero, and that the value of \( \gamma_2 \) be close to zero. In the range of interest rates observed, the zero-profit contour \( g(\cdot) \) is virtually perfectly inelastic. Since the likelihood function is undefined when \( \gamma_2 = 0 \), we restrict \( \gamma_2 \) to be near zero. This result is likely to be a consequence of our assumption that \( g(\cdot) \) is linear (non-linear estimates of \( g(\cdot) \) proved computationally unfeasible). Second, the \( t \)-ratios reported are all corrected for the presence of an estimated explanatory variable, namely the amount of credit obtained from the co-operative.  

Beginning with the demand side, we have two strong results. First, the estimate of the coefficient of the interest rate is negative and highly significant, and it implies an interest elasticity of demand of \(-0.22\) at the sample mean. Second, the coefficient of the estimated amount borrowed from the co-operative lies in the interval \([-1,0]\), as required by the theory. The hypothesis that households have constant absolute risk aversion is decisively rejected, for in that case the coefficient would be minus one.

Comparing the coefficients of the other regressors with their counterparts in the demand for co-operative credit, most are now insignificantly different from zero. Of those that are not, the coefficients on the district dummy variable and the value of land holdings have changed sign, the latter being a welcome

---

11 In addition to not being globally concave, this likelihood function exhibits the common feature in models with unobserved sample separation of being unbounded as certain correlations approach \( \pm 1 \). This is the case for the correlations involving \( \eta \), hence they are restricted to zero. It is also the case that the likelihood is unbounded as \( \rho_{\text{eq}} \) approaches \( \pm 1 \), but the highest interior maximum was found using a grid search. Sundberg (1974) shows that this interior maximum is consistent.

12 Murphy and Topel (1985) provide general methods.

13 If there is decreasing absolute risk aversion, the notional demand for co-operative credit will be somewhat more elastic.
Table 5
Informal market: parameter estimates and asymptotic t-ratios

<table>
<thead>
<tr>
<th>Variable</th>
<th>Demand Estimate</th>
<th>Demand t</th>
<th>Supply Estimate</th>
<th>Supply t</th>
<th>Tying index estimate</th>
<th>Tying index t</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>1.968</td>
<td>1.17</td>
<td>-3.806</td>
<td>-2.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant*tie = 1</td>
<td></td>
<td></td>
<td>3.414</td>
<td>2.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>village endowment</td>
<td>1.480</td>
<td>1.19</td>
<td>0.124</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>district</td>
<td>-9.191</td>
<td>-7.43</td>
<td>-6.331</td>
<td>-6.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>high caste</td>
<td>1.720</td>
<td>1.17</td>
<td>1.797</td>
<td>1.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>medium caste</td>
<td>-0.217</td>
<td>-0.16</td>
<td>-2.123</td>
<td>-2.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>visible assets</td>
<td></td>
<td></td>
<td>0.057</td>
<td>5.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>family size</td>
<td>-0.023</td>
<td>-0.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dependency ratio</td>
<td>-0.013</td>
<td>-1.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>working males in agriculture</td>
<td>-0.003</td>
<td>-0.16</td>
<td></td>
<td></td>
<td>0.002</td>
<td>0.12</td>
</tr>
<tr>
<td>liquid assets</td>
<td>0.005</td>
<td>1.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>value of draft power</td>
<td>0.004</td>
<td>0.89</td>
<td></td>
<td></td>
<td>0.008</td>
<td>0.44</td>
</tr>
<tr>
<td>land value</td>
<td>0.047</td>
<td>2.19</td>
<td></td>
<td></td>
<td>0.024</td>
<td>0.30</td>
</tr>
<tr>
<td>predicted amount of</td>
<td>-0.008</td>
<td>-0.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>regulated credit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Interactions with κ (shift from f(·) to D(·))

<table>
<thead>
<tr>
<th></th>
<th>Demand t</th>
<th>Supply t</th>
<th>Tying index t</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>-1.924</td>
<td>-1.19</td>
<td></td>
</tr>
<tr>
<td>district</td>
<td>24.303</td>
<td>7.09</td>
<td></td>
</tr>
<tr>
<td>high caste</td>
<td>-5.540</td>
<td>-1.79</td>
<td></td>
</tr>
<tr>
<td>land value</td>
<td>0.036</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>interest rate</td>
<td>-0.089</td>
<td>-4.68</td>
<td></td>
</tr>
<tr>
<td>δ + constant</td>
<td></td>
<td></td>
<td>-0.187</td>
</tr>
<tr>
<td>σ_u</td>
<td></td>
<td></td>
<td>14.45</td>
</tr>
<tr>
<td>σ_v</td>
<td>3.083</td>
<td>9.23</td>
<td></td>
</tr>
<tr>
<td>σ_w</td>
<td>0.813</td>
<td>10.73</td>
<td></td>
</tr>
</tbody>
</table>

κ = 196 - log 𝝋 = 493.66
Correlation between predicted and actual quantity transacted: 0.38
Correlation between predicted and actual interest rates: 0.20
Likelihood ratio test: The null hypothesis that all coefficients except the four constant terms (demand, tie, no tie, δ + constant), λ and the two variance terms (σ_u, σ_v) are zero: χ²(23) = 99.80; p = 0.00.

Improvement over the finding in Section 5.1. Although there are good grounds for the parameter vectors β and β to differ, the extent of the differences suggests to us that the results need to be interpreted with some caution. Finally, with the exception of the district dummy variable, the various components of the horizontal distance between f(·) and D₂(·) are not statistically significantly different from zero.

The estimates for the supply equation are very satisfactory. Most striking of all is the effect of tying credit transactions to crop marketing deals. The average increase in loan size as a result of an acceptance of a tie is about Rs. 7,220 (the difference between the two constant terms ×1,000), while the average size of all loans was approximately Rs. 4,600. Households in Jullunder and those of
Table 6

Predictive success of the model: informal market

<table>
<thead>
<tr>
<th>Observed status</th>
<th>Predicted status</th>
<th>Observed count</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Borrowing</td>
<td>Not borrowing</td>
</tr>
<tr>
<td>borrowing</td>
<td>61</td>
<td>20</td>
</tr>
<tr>
<td>not borrowing</td>
<td>48</td>
<td>67</td>
</tr>
<tr>
<td>predicted count</td>
<td>109</td>
<td>87</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Observed status</th>
<th>Predicted status</th>
<th>Observed count</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r &gt; 0 )</td>
<td>( r = 0 )</td>
</tr>
<tr>
<td>( r &gt; 0 )</td>
<td>43</td>
<td>14</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>30</td>
<td>109</td>
</tr>
<tr>
<td>predicted count</td>
<td>73</td>
<td>123</td>
</tr>
</tbody>
</table>

medium caste also faced less generous offer schedules of unregulated credit. Visible asset holdings increased the supply of credit, as expected, and the coefficient is highly significant.

Where the tying index is concerned, the specification of the supply equation makes it impossible to identify the coefficient \( \delta_1 \). Only the constant in the supply equation (16) was permitted to vary between tied and untied contracts, so that \( x_2 \alpha_2 \) is a number which is constant across the sample. It follows that the constant term of the index function, that is, the constant in \( \delta_0 \), cannot be estimated separately from \( \delta_1 \), and only a linear combination of the two can be identified. That the estimated coefficient of the constant plus \( \delta_1 \) is negative is not, therefore, inconsistent with there being benefits from tying in the form of more generous credit (\( \delta_1 > 0 \)). The coefficients of the predictors of crop output in the index function are insignificantly different from zero and have the wrong sign. This may indicate that the tying contract does not require pledging of the entire crop, but only of an adjustable portion. It could indicate that the decision to accept (or not) a tied contract is driven by considerations outside our model, or it could be a consequence of our unavoidably parsimonious specification of the supply of credit function \( g(\cdot) \), which includes no interaction terms. Finally, this result could be an artifact of the absence of information on commission rates. From interviews with commission agents we know that these rates are fairly uniform, but we have no individual data, and these rates would presumably affect the attractiveness of a proposed tying arrangement. In any case, our inability to identify statistically significant determinants of this decision is worrisome, given the quantitative importance of an agreement to tie on the size of the loan.

Turning to the overall performance of the model, the likelihood ratio test of the null hypothesis that all of the coefficients other than the constant and
variance terms are zero yields a \( \chi^2(23) \) test statistic of 100 \( (p = 0) \). Its predictive performance in assigning households correctly between the qualitative status of borrowing and not borrowing is summarized in Table 6. The number of households that borrowed is overestimated, as in the first-stage model. The overall success rate in assigning households correctly is 66%, compared with 52% for the naive model that assigns households with probabilities equal to the actual proportion of households appearing in each regime. The success rate in assigning households to the two regimes of positive or zero interest rates is 78%, compared with the naive model which achieves a success rate of 59%.

6. Concluding observations

This analysis of Punjabi cultivators' transactions in the markets for short-term credit has yielded four main conclusions. First, there was pervasive rationing in the regulated market, with many households failing to get offers of credit at all. Second, the elasticity of the notional demand for credit was low, the point estimate at the sample mean being \(-0.22\). Third, despite a substantial difference between the rates of interest on regulated and unregulated credit, pervasive rationing in the regulated market and inelastic demand made borrowing in the informal market an attractive option for many households. Thus, regulation led to a significant degree of market segmentation. Fourth, the cultivator's acceptance of a crop marketing tying condition greatly increased the amount of finance a private lender was prepared to offer him.

What implications can be drawn for public policy? In answering this question, one must keep in mind two qualifications concerning the methods used to arrive at the main conclusions. First, we have emphasized that if lenders can observe at least some of the characteristics of potential borrowers that affect contractual performance, then exclusion restrictions cannot be imposed to identify the supply side of the model. Second, the comparatively small sample and the complexity of the problem we have attempted to analyze require that one impose a good deal of structure to make estimation possible. The results must, therefore, be interpreted with caution.

At center stage is the regulated interest rate. Even allowing for differences in transactions costs, the gap in interest rates ruling in the two segments was quite large, which invites the proposal that the nominal gap be reduced. The finding that the demand for credit is interest-inelastic implies that even a substantial rise in the rate charged by co-operatives would not have much of an effect on the demand for co-operative credit. Hence, the main effect of a rise in the regulated rate would be to transfer some of the rents currently enjoyed by the co-operatives' borrowers back to the co-operatives themselves. As the average loan was about Rs. 3,500 per borrower, a rise in the regulated rate from 11% p.a. to, say, 16% p.a. would have produced a transfer of about Rs. 85 a season, or almost ten times the daily wage.

Where the incidence of an increase is concerned, existing poor borrowers would be worse off, and this rise in costs could induce some of them to lease
out more land (net) or even to give up cultivation altogether. Since we have shunned any attempt to incorporate the choice between own cultivation and leasing into an already complex econometric structure, we cannot provide a firm estimate of how large such an effect might be. However, our findings that most households were rationed in the regulated segment of the market and that wealth is the main determinant of household credit limits lead to the conclusion that the principal immediate losers would be the wealthy. Thus, while some of the co-operatives' poorer members would be hurt, it is hard to make the case that the impact of an increase in the regulated interest rate would be broadly regressive.\textsuperscript{14} Despite the caution we sounded earlier, our findings place an additional onus of justification upon those who would soldier on with the policy of subsidizing rural credit. In any event, what an increase in the regulated rate from 11\% p.a. to 16\% p.a. will not do is to produce much of an effect on the level of excess demand for funds. In view of the informational problems that beset regulated lenders, this conclusion should not be wholly surprising.

Turning to the private segment of the market, the fact that many such loans are tied to commodity marketing implies that regulating the marketing system may impinge on the credit markets. About 35\% of the sample, and 85\% of those borrowing from private sources, have such tied loans. Now, the estimate of the amount by which a tied transaction raises the supply of private credit to a household is Rs. 7,220. Thus, by taking the minimum of Rs. 7,220 and the amount actually transacted in the private market for each of the households having tied loans, we arrive at an estimate of the amount by which the total volume of transactions in the unregulated market would fall in the event that such tying arrangements were banned. On this basis, the volume of private credit advanced to cultivators would fall by over 70\%; the corresponding fall in the overall volume of credit would be over 35\%, since the total amount of private credit is slightly larger than that advanced by co-operatives.

In view of the dependence of Punjab's agriculture on credit, these are rather sobering figures. They suggest that successful attempts by the government to force private traders and commission agents out of the marketing business will produce significant adverse effects on agricultural output unless the regulated market can begin to function in such a way as to substitute for the consequent withdrawal of private credit. Wholesale reform of the regulated credit market, however, is a topic far beyond the scope of the present paper.

\textbf{ACKNOWLEDGEMENTS}

We are indebted to Jim Heckman, Anand Swamy, participants in seminars at Berkeley, Boston University, Heidelberg, Indiana, Irvine, the LSE, Pennsylvania, USC, and Yale, and especially to two anonymous referees for helpful comments. Bell gratefully acknowledges financial support from

\textsuperscript{14} One of the referees pointed out that if, instead of raising the interest rate on co-operative loans, loanable funds could be increased, poor peasants would not be hurt, and to the extent this increase lowers the interest rate in private loan markets, those who borrow in the private markets will benefit as well.
the Alexander von Humboldt Foundation, and Udry from the NSF. We are also grateful to the World Bank for funding the survey under RPO-671-89. We retain sole responsibility for all surviving errors.

REFERENCES


APPENDIX

To establish our first claim, namely, that $V_t$ may possess an interior maximum at $(Q_t^*, \theta_t^*)$, we assume the following:

**Assumption 1.** $U(Y)$ is strictly concave for $Y > 0$ and $U(Y) = -\infty$ for $Y \leq 0$; $\lim_{Y \to 0} U'(Y) = \infty$ and $\lim_{Y \to -\infty} U'(Y) = 0$. 

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RATIONING, SPILLOVER, AND INTERLINKING

Assumption 2. \( F(L, N; \bar{H}) \) is strictly concave in \( L \) and \( N \), and satisfies the Inada conditions, \( \lim_{L \to 0} (\partial F/\partial L) = \infty \) and \( \lim_{L \to -\infty} (\partial F/\partial L) = 0 \), with analogous conditions holding for \( N \).

Assumption 3. \( \theta \) is continuously distributed with density \( h(\theta) \) on the support \([0, \infty)\). \( h(0) > 0 \).

Define

\[
G(K, \bar{H}) \equiv \max_{L, N} F(L, N; \bar{H}) \tag{A.1}
\]

subject to

\[
wL + pN = K \tag{A.2}
\]

Given Assumption 2, (i) there exist a unique \( L(w, p, K, \bar{H}) \) and \( N(w, p, K, \bar{H}) \) that maximizes \( F \); (ii) \( G \) is strictly concave in \( K \); and (iii) \( G \) satisfies the Inada conditions.

In the absence of borrowing, \( K = wL \). The household sets \( L = L(w, p, \bar{w}, \bar{H}) \) and \( N(w, p, \bar{w}, \bar{H}) \) every period and realizes the life-time expected utility

\[
V_D = EU[\theta G(wL, \bar{H}) + \bar{Y}] \tag{A.3}
\]

With borrowing, \( K = wL + Q_1 \). Now

\[
V_1(\theta_1) = \left\{ \int_0^\infty U(Y_D)h(\theta)d\theta + \int_0^\infty U(Y_D - (1 + r_1)Q_1)h(\theta)d\theta + \delta V_D \right\} \left( 1 - (1 - \Delta_1)\delta \right)
\]

if \( 1 > \Delta_1 > 0 \),

\[
V_1 = \int_0^\infty U(Y_D)h(\theta)d\theta + \delta V_D \text{ if } \Delta_1 = 1 \tag{A.4}
\]

where

\[
Y_D = \theta G(K; \bar{H}) + \bar{Y} \tag{A.5}
\]

and

\[
\Delta_1 = \int_0^\theta h(\theta)d\theta \tag{A.6}
\]

Several observations are in order. First, \( Y_D \) and \( Y_D - (1 + r_1)Q_1 \) are both strictly concave in \( Q_1 \). By Assumption 1, therefore, \( V_1 \) is strictly concave in \( Q_1 \) given \( \theta_1 \). Second, by borrowing \( Q_1 > 0 \), consuming \( Y_D \), and defaulting, the household can assure itself a life-time utility of

\[
V_1(Q_1, \infty) = \int_0^\infty U(Y_D)h(\theta)d\theta + \delta V_D \tag{A.7}
\]

Since \( Y_D \) is monotone increasing in \( Q_1 \), \( V_1(Q_1, \infty) \) is monotone increasing in \( Q_1 \) as well. This means that by borrowing an indefinitely large amount and defaulting, the household can achieve a life-time utility of \( \lim_{Q_1 \to \infty} V_1(Q_1, \infty) \leq \infty \), which is unbounded if \( U(Y) \) is unbounded.

Third, there is the ceiling imposed by the co-operative, say \( Q_f \). By taking \( Q_f \) and always defaulting, the household can achieve a life-time utility of \( V_1(Q_f, \infty) \). This is not, however, the best the household can do. By maximizing \( V_1 \) subject to \( Q_1 < Q_f \) and choosing some default probability \( \Delta_1 < 1 \) (i.e. \( \theta_1 < \infty \)), the maximum \( V_1 \) with respect to \( Q_1 \) and \( \theta_1 \) will occur either at

\[
Q_1 = Q_f, \theta_1 = \theta_f < \infty \tag{A.8}
\]

or at

\[
Q_1 = Q_f < Q_f, \theta_1 = \theta_f < \infty \tag{A.9}
\]

In either case, \( \max V_1 > V_1(Q_f, \infty) \). The notional demand which is referred to as \( D_f \) in the text corresponds to \( Q_f \) of (A.9).

In order to prove the above characterization of the optimum, note that since (A.9) represents an interior maximum for \( V_1 \), the necessary first-order conditions are

\[
\frac{\partial V_1}{\partial \theta_1}\bigg|_{\theta_1 = \theta_f} = \left[ \frac{\delta(V_1 - V_D) + U(Y_D(\theta_f) - (1 + r)Q_f)}{1 - \Delta_1(1 - \delta)} \right] h(\theta_f) = 0 \tag{A.10}
\]
and
\[ \frac{\partial V_1}{\partial Q_1} \bigg|_{Q_1 = \theta_1} \equiv \left[ 1 - \Delta_3 (1 - \delta) \right] \int_0^\infty U'(Y_D) \frac{\partial G}{\partial Q_1} \theta h(\theta) d\theta \\
+ \int_0^\infty U'[Y_D - (1 + r_1)Q_1] [\theta \frac{\partial G}{\partial Q_1} - (1 + r_1)h(\theta)] d\theta = 0 \] 

(A.11)

Both these are easily interpreted. (A.10) says that at \( \theta_1 = \theta_1^0 \), the gain from defaulting over repaying, i.e. \( \{U[Y_D(\theta_1^0)] + \delta V_D\} - \{U[Y_D(\theta_1^0) - (1 + r_1)Q_1] + \delta V_1\} \) is zero. (A.11) says that at \( Q_1 = Q_1^0 \), the marginal welfare gain from the extra output \( \partial G/\partial Q_1 \), viz.
\[ \int_0^\infty U'[Y_D - (1 + r_1)Q_1^0] \frac{\partial G}{\partial Q_1} \theta h(\theta) d\theta + \int_0^\infty U'[Y_D - (1 + r_1)Q_1^0] \frac{\partial G}{\partial Q_1} \theta h(\theta) d\theta \]

just equals the marginal welfare loss from repaying the loan whenever repayment occurs, viz
\[ (1 + r_1) \int_0^\infty U'[Y_D - (1 + r_1)Q_1^0] h(\theta) d\theta \]

By Assumption 1, \( U[Y_D(\theta_1^0)] - \{U[Y_D(\theta_1^0) - (1 + r_1)Q_1^0]\} \) tends to zero as \( \theta_1 \) becomes unboundedly large for any finite \( Q_1 \). At \( Q_1 = Q_1^0 \), therefore, the household will certainly choose \( \Delta_3 < 1 \). An interior maximum of \( V_1 \) exists if (A.10) and (A.11) possess a solution with \( Q_1 > Q_1^0 \).

Fourth, the role of the ceiling \( Q_1^0 \), even in case (A.9), is clear. Even though \( \max V_1 \equiv V_1(Q_1^0, \theta_1^0) > V_1(Q_1^1, \infty) \), the fact that \( V_1(Q_1^1, \infty) < \lim_{Q_1 \to \infty} V_1(Q_1^1, \infty) \) implies one cannot rule out \( V_1(Q_1^0, \theta_1^0) \). As such, if there were no ceiling, the household would not choose \( (Q_1^0, \theta_1^0) \) but instead opt to default after taking an unbounded loan. The ceiling \( Q_1^0 \) prevents this from happening.

The second claim is that the map of lifetime expected utility contours in \((Q, r)-space\) is different under an exclusive private loan. We have
\[ V_2 = \left\{ \int_0^\infty [\bar{h}(\theta) \theta h(\theta) d\theta + \int_0^\infty U[Y_D - (1 + r_2)Q_2] h(\theta) d\theta + \Delta_2 \delta V_2] \right\} / [1 - (1 - \Delta_3) \delta] \] 

(A.12)

where
\[ \theta_2 \equiv [(1 + r_2)Q_2]/\left[ G[1 + Q_2; \bar{H}] \right] \] 

(A.13)

In this case, the household has effectively just the single choice variable \( Q_2 \). A comparison of (A.4) with (A.12) reveals that \( V_1 \neq V_2 \) where \( (Q_1, r_1) = (Q_2, r_2) \), even if, by fluke, \( \theta_1 = \theta_2 \) . The maps of iso-\( V_1 \) and iso-\( V_2 \) are also different; for the first-order condition w.r.t. \( Q_2 \) contains additional terms arising from (A.13), and \( Y = \bar{Y} \forall \theta < \theta_2 \). Hence, \( Q_2^0(r) \neq Q_2^0(r) \).

To conclude, we establish that \( g(Q_2; \cdot) \) is upward-sloping. Differentiating (9) totally for given \( \theta_2 \) and rearranging, we obtain
\[ \frac{dr_2}{dQ_2} = -\left\{ \int_0^\infty [\theta \frac{\partial h}{\partial Q_2} - (1 + r_2) h(\theta) d\theta + (r_2 - r_0) \right\} / (1 - \Delta_2) Q_2 \] 

(A.14)

By setting \( E \theta \) in (9) to zero, solving for \( r_2 - r_0 \) and substituting in (A.14), one gets
\[ \frac{dr_2}{dQ_2} = -\int_0^\infty \left[ \theta \frac{\partial G}{\partial Q_2} - \frac{G}{Q_2} \right] h(\theta) d\theta / (1 - \Delta_2) Q_2 \]

The strict concavity of \( G \) with respect to \( Q_2 \) implies \( G/Q_2 > \partial G/\partial Q_2 \). Hence \( dr_2/dQ_2 > 0 \forall Q_2 > 0 \).