Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity

by

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Introduction:

Three sources of gains from trade in monopolistic competition model:

1) Expansion in product variety (Krugman, 1979)
   • but only if the imported varieties do not eliminate a commensurate amount of domestic varieties

2) Pro-competitive effect due to reduction in markups (Krugman, 1979)
   • But this is a social gain since reduced markups leads firms to expand scale, since \( \frac{P}{MC} = \frac{AC}{MC} \)

3) Selection of more efficient firms into exporting (Melitz, 2003)

Recent literature on heterogeneous firms has emphasized the selection effect:
• Melitz + Pareto (Chaney, 2008) means that the welfare cost due to the reduction in domestic varieties with trade just offsets the welfare gain due to increase import variety, so that the variety channel does not operate (Feenstra, 2010) (and pro-competitive channel does not operate due to CES)

• This explains why the gains from trade formula in Arkolakis, Costinot, Rodriguez-Clare (ACR, 2012) for the Melitz-Chaney model depends only on the Pareto parameter, and not on the CES elasticity

\[
d \ln W = - \frac{d \ln \lambda}{\theta}, \quad \lambda = \text{share of expenditure on domestic products}
\]

• Melitz and Redding (2013) argue that despite this formula, the gains from trade are higher with heterogeneous than homogeneous firms because in the former case allows for self-selection of firms into exporting

• Arkolakis, Costinot, Donaldson and Rodriguez-Clare (ACDR, 2012) obtain the same formula even with a broad class of preferences allowing for non-constant markups by firms

• But the variety and pro-competitive effects do not operate in ACDR when using a Pareto distribution with support that is unbounded above
**Intuition:** For why pro-competitive effect vanishes with unbounded Pareto distribution (with homothetic demand):

*Measure Markups as the ratio (not difference) between price and MC:*

- lowest productivity domestic firm has Markup ratio = 1
- highest productivity domestic firm has Markup ratio = \(\infty\)
- So range is \([1, \infty)\), with distribution *within* this range being Pareto
- This also holds for foreign firms even though MC include trade costs!
- So the distribution of markups is identical for home and foreign firms, and is not affected by trade costs
- Changes in trade costs only affects the *extensive margin* of foreign firms, i.e. the mass of firms selling within the range \([1, \infty)\)
- Clearly not true with *bounded* Pareto, in which case this range has a finite and endogenous upper-bound; this bound changes on the *intensive margin*
**Contribution of this paper:**

- Derive effects of trade liberalization in a Melitz-style model with bounded Pareto distribution of productivities

  Motivation: Helpman, Melitz and Rubenstein (2008)
  Sutton (2012) “you can’t make something out of nothing”

- Use quadratic mean of order $r$ (QMOR) preferences due to Diewert (1976)

**Results:**

- Find that all three sources of gains from trade operate only if the Pareto distribution has a finite upper bound for productivities

- But also shown that for the types of trade liberalization considered, the ACR formula continues to hold as an *upper bound* to the welfare gains
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Consumers
QMOR expenditure function
Properties of demand
Compare with translog and with ACDR
Decomposition of welfare

Firms
Bounded Pareto distribution
Autarky Equilibrium

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Conclusions
Consumers:

Quadratic mean of order $r$ (QMOR) expenditure function (Diewert, 1976)

$$\left[ \sum_i \sum_j b_{ij} p_i^{r/2} p_j^{r/2} \right]^{1/r}, \quad r \neq 0,$$

Symmetric case where $b_{ii} = \alpha$, $b_{ij} = \beta$ for $i \neq j$, and a continuum of goods:

$$e_r(p) = \left[ \alpha \int p_\omega^r d\omega + \beta \left( \int p_\omega^{r/2} d\omega \right)^2 \right]^{1/r}, \quad r \neq 0, \quad \tilde{N} \equiv \int d\omega$$

Cost of obtaining one unit of utility (homothetic preferences), Cost of living.

Cases:

(a) CES: $\alpha > 0$, $\beta = 0$, $r = 1 - \sigma < 0$

(b) Translog: $r \to 0$ 

$$\ln e_0(p) = \frac{1}{\tilde{N}} \int \ln p_\omega d\omega - \frac{\gamma}{2\tilde{N}} \int \int \ln p_\omega (\ln p_\omega - \ln p_\omega') d\omega d\omega'$$

(c) Generalized Leontief: $r = 1$

(d) Quadratic: $r = 2$
**Assumption 1**

(a) If $r < 0$ then $\alpha > 0$, $\beta < 0$ and $[\tilde{N} + (\alpha / \beta)] < 0$ ;

(b) If $r > 0$ then $\alpha < 0$, $\beta > 0$ and $0 < [\tilde{N} + (\alpha / \beta)] < N$ ;

(c) As $r \to 0$ then $\alpha = \left(\frac{1}{\tilde{N}} - \frac{2\gamma}{r}\right)$ and $\beta = \frac{2\gamma}{r\tilde{N}}$ for any $\gamma > 0$.

Only *available* goods with prices $< p^*$ are purchased, $\Omega \equiv \{\omega \mid p_\omega \leq p^*\}$:

$$p^* = \left(\frac{N}{N - [\tilde{N} + (\alpha / \beta)]}\right)^{2/r} \left(\int_{\Omega} \frac{1}{N} p_\omega^{r/2} d\omega\right)^{2/r}, \text{ with } 0 < N \equiv \int_{\Omega} d\omega < \tilde{N}$$

**Proposition 1**

Under Assumption 1, for $N > 0$ and $r \leq 2$ the QMOR expenditure function is globally positive, non-decreasing, homogeneous of degree one and concave in prices, with a finite reservation price.
Four properties of demand:

1. \( q_{\omega}(p) = \alpha u \left[ \frac{p_{\omega}}{e_r(p)} \right]^{r-1} \left[ 1 - \left( \frac{p^*}{p_{\omega}} \right)^{r/2} \right] \rightarrow \text{CES as } p^* \rightarrow \infty \)

2. \( \eta_{\omega} = -\frac{\partial \ln q_{\omega}}{\partial \ln p_{\omega}} = 1 - r + \frac{r}{2} \left\{ \left( \frac{p^*}{p_{\omega}} \right)^{r/2} \left[ \left( \frac{p^*}{p_{\omega}} \right)^{r/2} - 1 \right] \right\} > 0 \)

3. \( \frac{\partial \eta_{\omega}}{\partial \ln(p_{\omega} / p^*)} = (\eta_{\omega} - 1 + r)(\eta_{\omega} - 1 + \frac{r}{2}) > 0 \) Increasing in price

4. \( s_{\omega}(p) = \frac{p_{\omega}q_{\omega}(p)}{w} = \frac{d(p_{\omega} / p^*)}{D(p)} \), with \( d\left( \frac{p_{\omega}}{p^*} \right) = \alpha \left( \frac{p_{\omega}}{p^*} \right)^r \left[ 1 - \left( \frac{p^*}{p_{\omega}} \right)^{r/2} \right] \)

\( D(p) \equiv \int_\Omega d(p_{\omega} / p^*)d\omega \)

ACDR only use a function like \( d \) to define shares, without integrating using \( D \) because \( \int_\Omega d(p_{\omega} / p^*)d\omega = 1 \) automatically, e.g. translog.
Final property:

Replacing prices for goods not available by their reservation price:

\[ e_r(p) = p^* D(p)^{1/r} \]

Define the “adjusted” demand shares:

\[ z_\omega(p) \equiv \frac{s_\omega(p)(p^*/p_\omega)^{r/2}}{\int_\Omega s_\omega'(p)(p^*/p_\omega)^{r/2} d\omega}, \text{ and } H \equiv \int_\Omega z_\omega(p)^2 d\omega \]

Then, \[ D(p)^{1/r} = \left[ -\alpha \left( \tilde{N} + \frac{\alpha}{\beta} \right) \right]^{1/r} \left[ 1 - \left( \tilde{N} + \frac{\alpha}{\beta} \right) H \right]^{1/r} \]

If \( p^* \) falls then the cost of living is reduced, but this is offset if the Herfindahl also falls (due to variety increasing).

- Later decompose \( p^* \) into variety, and firms’ average markups and costs
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**Conclusions**
**Firms:**

Labor is the only input, so with zero expected profits, Welfare = $w/E_r(p)$.

As in Melitz (2003), firms receive a random draw of productivity denoted by $\varphi$, so marginal costs are $a/\varphi$, $a = \text{labor requirement}$.

**Assumption 2**

(a) The productivity distribution is Pareto, $G(\varphi) = (1 - \varphi^{-\theta}) / (1 - b^{-\theta})$, $1 \leq \varphi \leq b$, where the upper bound is $b \in (1, +\infty]$ (bounded or unbounded), $\theta > \max\{0, -r\}$;

(b) There is a sunk cost $F$ of obtaining a productivity draw, but no fixed cost of production.

We follow ACDR and let $\mu = p / (a / \varphi)$ denote the ratio of price to MC, while $\nu = p^* / (a / \varphi)$ denotes the ratio of the reservation price to MC.
Markups $\mu$ are solved uniquely from demand elasticity as:

$$\mu = \frac{\eta(\mu / \nu)}{\eta(\mu / \nu) - 1} \Rightarrow \text{Sol'n } \mu(\nu) \text{ with } 0 < \frac{\nu \mu'(\nu)}{\mu(\nu)} < 1 \quad \text{Partial pass-through}$$

The change in variables from $\phi$ to $\nu$, $\nu = p^*/(a / \phi)$, leads to the decomposition:

**Lemma**

The reservation price in the closed economy is:

$$p^* = \left(\frac{N}{N - [\tilde{N} + (\alpha / \beta)]}\right)^{2/r} \frac{\tilde{g}(\nu)}{\tilde{G}(\nu^*)} \frac{\int_1^{\nu^*} \mu(\nu)^{r/2} \tilde{g}(\nu) d\nu}{\int_1^{\nu^*} \frac{G(\nu^*)}{G(\nu^*)} \tilde{g}(\nu) d\nu}$$

where $\tilde{g}(\nu) = g(\nu) / \nu^{r/2}$ is an “adjusted” density and the upper bound for $\nu$, denoted by $\nu^*$, for most productive firm, is:

$$\nu^* = \frac{bp^*}{a} \rightarrow \infty \text{ as } b \rightarrow \infty$$

Intensive margin \hspace{1em} No intensive margin
Autarky Equilibrium conditions:

1. Free entry/zero expected profit:

\[
F = \int_{1}^{v^*} \left[ \frac{\mu(v) - 1}{\mu(v)} \right] \frac{L d\left( \frac{\mu(v)}{v} \right)}{D(p)} \left( \frac{p^*}{a} \right)^\theta g(v) dv = \frac{L \int_{1}^{v^*} \left[ \frac{\mu(v) - 1}{\mu(v)} \right] d\left( \frac{\mu(v)}{v} \right) \left( \frac{p^*}{a} \right)^\theta g(v) dv}{N_e \int_{1}^{v^*} d\left( \frac{\mu(v)}{v} \right) \left( \frac{p^*}{a} \right)^\theta g(v) dv}
\]

2. Surviving firms:

\[
N = N_e \int_{1}^{v^*} \left( \frac{p^*}{a} \right)^\theta g(v) dv = N_e \left( \frac{p^*}{a} \right)^\theta G(v^*),
\]

3. Reservation price:

\[
N - \left( \tilde{N} + \frac{\alpha}{\beta} \right) = N_e \int_{1}^{v^*} \left( \frac{\mu(v)}{v} \right)^{r/2} \left( \frac{p^*}{a} \right)^\theta g(v) dv.
\]
Autarky Equilibrium conditions with unbounded Pareto:

1. Free entry/zero expected profit:

\[ F = \int_1^\infty \left[ \frac{\mu(v) - 1}{\mu(v)} \right] \frac{Ld}{\mu(v)} \left( \frac{\mu(v)}{v} \right) \left( \frac{p^*}{a} \right)^\theta g(v) dv = \frac{L \int_1^\infty \left[ \frac{\mu(v) - 1}{\mu(v)} \right] d \left( \frac{\mu(v)}{v} \right) \left( \frac{p^*}{a} \right)^\theta g(v) dv}{N_e \int_1^\infty d \left( \frac{\mu(v)}{v} \right) \left( \frac{p^*}{a} \right)^\theta g(v) dv} \]

- Solve for \( N_e \) as linear in \( L \)

2, 3. Surviving firms / Reservation Price:

\[ \frac{N}{N - \left( \tilde{N} + \frac{\alpha}{\beta} \right)} = \frac{N_e \left( \frac{p^*}{a} \right)^\theta G(\infty)}{\left( \frac{N_e \int_1^\infty \left( \frac{\mu(v)}{v} \right)^{r/2} \left( \frac{p^*}{a} \right)^\theta g(v) dv}{N_e \int_1^\infty d \left( \frac{\mu(v)}{v} \right) \left( \frac{p^*}{a} \right)^\theta g(v) dv} \right) \]

- Solve for \( N \) independent of \( L \) (due to strong selection of firms)!
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Frictionless Trade:

Consider growth in country size $L$ (due to frictionless trade):

$$d \ln N_e = \left( \frac{1+A}{1+A+B} \right) d \ln L \quad \text{and} \quad d \ln p^* = \frac{-d \ln L}{\theta (1+A+B)}$$

$$A \equiv \frac{N_e}{[\tilde{N} + (\alpha / \beta)]} \left( \frac{b^{-\theta}}{1-b^{-\theta}} \right) \left[ 1 - \left( \frac{\mu(v^*)}{v^*} \right)^{r/2} \right] = 0 \text{ for } b = \infty, > 0 \text{ for } b < \infty$$

$$B \equiv \left[ \frac{L}{F} \left( \frac{\mu(v^*) - 1}{\mu(v^*)} \right) - N_e \right] \frac{d[\mu(v^*) / v^*] b^{-\theta}}{D(p)(1-b^{-\theta})} = 0 \text{ for } b = \infty, > 0 \text{ for } b < \infty$$

Unbounded Pareto: $d \ln N_e = d \ln L$, $d \ln p^* = -\frac{d \ln L}{\theta}$, so $N = N_e \left( \frac{p^*}{a} \right)^\theta$

is fixed, as is the Herfindahl index $H$
**Proposition 3**

Under Assumptions 1 and 2, an increase in country size $L$ under frictionless trade leads to: (a) when $b = \infty$, then $p^*$ falls only due to the drop in the average of firm costs, with the Herfindahl index $H$ fixed;

Let: $\lambda = \text{share of consumption coming from domestic production}$

Denote: $\bar{L} = \text{domestic labor force}$ by, $L = \text{world labor force} > \bar{L}$,

Then: $\lambda = \bar{L} / L$ with frictionless trade $\Rightarrow d \ln \lambda = -d \ln L < 0$

$\Rightarrow$ from $d \ln p^* = -\frac{d \ln L}{\theta}$ we obtain the ACR/ACDR formula,

$$d \ln(\text{Welfare}) = -d \ln p^* = \frac{d \ln \lambda}{\theta}$$
**Bounded Pareto:** It is immediate that $A > 0$ and also $B > 0$ for $b < \infty$, because Lerner index takes on its highest value at $v^*$ (most efficient firm):

$$B \propto \left( \frac{\mu(v^*) - 1}{\mu(v^*)} \right) - \int_1^{v^*} \left( \frac{\mu(v) - 1}{\mu(v)} \right) \left[ \frac{d\left( \frac{\mu(v)}{v} \right)g(v)}{\int_1^{v^*} d\left( \frac{\mu(v')}{v'} \right)g(v')dv'} \right] dv > 0$$

$$d \ln N_{e} = \left( \frac{1 + A}{1 + A + B} \right) d \ln L < d \ln L \quad \text{and} \quad d \ln p^* = \frac{-d \ln L}{\theta(1 + A + B)} > -\frac{d \ln L}{\theta}$$

*Less entry but less selection, so opposing effects on $N$; turns out that $N \uparrow$ and $H \downarrow$*

**Proposition 3**

Under Assumptions 1 and 2, an increase in $L$ under frictionless trade leads to:

(b) *when $b < \infty$, then variety $N$ rises, the Herfindahl falls, and the average of firm costs and markups fall;*
Average markup is falling because we are excluding the highest markups in:

\[
\left[ \int_1^{v^*} \mu(v)^{r/2} \frac{\tilde{g}(v)}{\tilde{G}(v^*)} \right]^{-2/r} \quad \text{as } v^* = \frac{bp^*}{a} \text{ falls (but not when } v^* = \infty)\]

But because variety \( N \) increases, the Herfindahl falls (crowding) so expenditure falls by less than the fall in the reservation price:

\[
e_r(p) = \frac{p^* D(p)^{1/r}}{\text{Variety} \times \text{Markup} \times \text{Costs} \times \text{Herfindahl}} \]

**Proposition 3**

Under Assumptions 1 and 2, an increase in \( L \) under frictionless trade leads to:

(c) the proportional welfare gain when \( b < \infty \) is less than that with \( b = \infty \).
Variable Trade Costs

- Restrict attention to symmetric equilibria
- In bounded case, consider small changes around frictionless equilibria
Variable Trade Costs

- Restrict attention to symmetric equilibria
- In bounded case, consider small changes around frictionless equilibria

**Proposition 4**

Under Assumptions 1–3, a small reduction in trade costs implies the following whether productivity is unbounded or is bounded with the change evaluated at the frictionless equilibrium: (a) no change in the mass of entrants $M_e$, the mass of varieties $N$, or the Herfindahl index $H$;
Variable Trade Costs

- Restrict attention to symmetric equilibria
- In bounded case, consider small changes around frictionless equilibria

**Proposition 4**
Under Assumptions 1–3, a small reduction in trade costs implies the following whether productivity is unbounded or is bounded with the change evaluated at the frictionless equilibrium: (a) no change in the mass of entrants $M_e$, the mass of varieties $N$, or the Herfindahl index $H$; (b) the same proportionate fall in the reservation price and rise in welfare of $-(1 - \lambda)d\ln\tau_0$, due to selection only.

Note that in the bounded case: reduced domestic markups just cancel with increased import markups around frictionless equilibrium.
So does anything else differ when productivities are bounded?

- For very large change in trade costs (from autarky), Proposition 3 applies.

- Also the **change in domestic variety** differs in the bounded case:

\[
M = M_e \int_1^{v^*} \left( \frac{p^*}{a} \right)^\theta g(v) dv = M_e \left( \frac{p^*}{a} \right)^\theta G(v^*)
\]

\[
d \ln M = \left[ \theta + \frac{g(v^*)v^*}{G(v^*)} \right] d \ln p^* \Rightarrow \begin{cases} 
  d \ln M = \theta (1 - \lambda) d \ln \tau_0 < 0 & \text{if } b = \infty \\
  d \ln M < \theta (1 - \lambda) d \ln \tau_0 < 0 & \text{if } b < \infty
\end{cases}
\]

At the frictionless equilibrium, **find that the \( \lambda \) share falls more with \( b < \infty \).**
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- For very large change in trade costs (from autarky), Proposition 3 applies.
- Also the change in domestic variety differs for the bounded case:

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d \ln M = \left[ \theta + \frac{g(v^*) v^*}{G(v^*)} \right] d \ln p^* \Rightarrow \begin{cases} 
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    d \ln M < \theta (1 - \lambda) d \ln \tau_0 < 0 & \text{if } b < \infty
\end{cases}
\]

At the frictionless equilibrium, find that the \( \lambda \) share falls more with \( b < \infty \)

**Corollary**

The gain from a small reduction in trade costs equals \(-d \ln \lambda / \theta > 0\) with an unbounded Pareto distribution for productivity, but is strictly less than this amount with a bounded Pareto distribution when evaluated at the frictionless equilibrium.
Conclusions:

Goal is to:

- show why product variety and pro-competitive effects vanish with Pareto distribution with unbounded support, leaving only the selection effect
- show how these results are restored when using bounded productivity

These results are not sensitive to the demand side, where we have used QMOR.

Explored two cases of trade liberalization:

- large reduction in tariffs from autarky to frictionless trade (like growth)
  
  The selection, variety and pro-competitive effects all operate, with bounded prod.

- small reduction in tariffs (with unbounded prod.) and evaluated at the frictionless equilibrium (with bounded prod.)
  
  The variety and pro-competitive effects do not operate, leaving only selection.

So variety and pro-competitive effects depend on being away from frictionless trade or on country growth.
We have also found that in both cases, the ACR/ACDR formula for the total gains from trade act as a \((strict)\) upper bound to that obtained in bounded case.

**Directions for further work:**

- Have not allowed for fixed costs of production or exporting
  
  *That would be enough to restore role for product variety and markups, because lower-bound of integration is endogenous. This is simplified in the CES and translog cases. Would be of interest to allow these fixed costs to fall, leading to more trade.*

- Have not explored any productivity distribution other than Pareto
  
  *Expect that the unbounded Pareto is the only distribution with the special feature that selection becomes the only operative force in the gains from trade.*