Technological Spillovers and Dynamics of Comparative Advantage

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**Motivation**

- Comparative advantage is **dynamic**: Korea – from rice to microchips.
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- Industrial policy to establish new sectors and **shape specialization**: semiconductors in 1980s and the “green energy” sector in 2000s.
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▶ Rationale: economies of scale and **technological spillovers**.
Motivation

- Comparative advantage is dynamic: Korea – from rice to microchips.

- Industrial policy to establish new sectors and shape specialization: semiconductors in 1980s and the “green energy” sector in 2000s.

- Rationale: economies of scale and technological spillovers.

- Modeling the dynamics of comparative advantage with technological linkages – a big open question. Challenge #1 is multiplicity of equilibria.
QUESTIONS FOR THIS PAPER

▶ How does comparative advantage evolve under the presence of intra- and inter-sector spillovers?
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▶ What is the socially optimal balanced growth path?
LITERATURE

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- **Empirical analysis** of spillovers: Elisson et. al. (2010), Greenstone et. al. (2010).
OUTLINE OF THE PAPER

1. Dynamic trade model with inter-sector spillovers.
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2. Description of balanced growth paths.
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1. **Dynamic trade model** with inter-sector spillovers.

2. Description of **balanced growth paths**.

3. **Industrial policy**.

4. **Calibration** of the model.

5. **Optimal policy**.
OUTLINE OF THE MODEL

1. **Trade part**: static, multi-sector EK (2002)

   - Discrete number of sectors and countries, continuum of varieties within each sector.

   - CES preferences at lower level, Cobb-Douglas at higher level.

   - Perfect competition.
Outline of the Model

   - Discrete number of sectors and countries, continuum of varieties within each sector.
   - CES preferences at lower level, Cobb-Douglas at higher level.
   - Perfect competition.

2. Evolution of technology part: dynamic
   - Learning-by-doing.
   - Accumulation of technologies through intra- and inter-sector spillovers.
   - Exogenous population growth.
TRADE PART: DEMAND

► Utility:

\[ U_j = \prod_{s=1}^{S} C_{j,s}^{\alpha_s} \]

\[ C_{j,s}^{\frac{\sigma-1}{\sigma}} = \int_{0}^{1} c_{j,s}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega, \]

where \( j \) – country, \( s \) – sector, \( \omega \) – variety within a sector.
TRADE PART: DEMAND

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➢ Expenditures:

\[ X_{j,s}(\omega) = \alpha_s L_j w_j \left( \frac{p_{j,s}(\omega)}{P_{j,s}} \right)^{1-\sigma}, \]
TRADE PART: DEMAND

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- Expenditures:

\[ X_{j,s}(\omega) = \alpha_s L_j w_j \left( \frac{p_{j,s}(\omega)}{P_{j,s}} \right)^{1-\sigma}, \]

- Prices:

\[ p_{j,s}(\omega) = \min_{k \in 1, \ldots, N} \{ p_{kj,s}(\omega) \}, \quad P_{j,s}^{1-\sigma} = \int_{0}^{1} p_{j,s}^{1-\sigma}(\omega) d\omega. \]
TRADE PART: SUPPLY

- Prices are equal to marginal costs:

\[ p_{ij,s}(\omega) = \frac{w_i d_{ij,s}}{z_{i,s}(\omega)} \]
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- Productivity of each technology is drawn from Pareto:
  \[ Pr(Q \leq q) = 1 - q^{-\theta}. \]
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- Productivity of each technology is drawn from \textit{Pareto}:
  
  \[ Pr(Q \leq q) = 1 - q^{-\theta}. \]

- Productivity \( z_{i,s}(\omega) \) is distributed \textit{Frechet}:
  
  \[ Pr(z_{i,s}(\omega) \leq z) = e^{-T_{i,s}z^{-\theta}}. \]
TRADE PART: STATIC EQUILIBRIUM

Given a vector of productivities $\{T_{i,s}\}_{i \in 1,\ldots,N}$ and sizes of countries $\{L_i\}_{i \in 1,\ldots,N}$ solve the system for sector labor demand $\{L_{i,s}\}_{i \in 1,\ldots,N}$ and wages $\{w_i\}_{i \in 1,\ldots,N}$:

$$L_{i,s}w_i = \sum_{j=1}^{N} \pi_{ij,s} \alpha_{s} w_j L_j \quad \forall i, s,$$

where $\pi_{ij,s} \equiv X_{ij,s} X_{j,s} = T_{i,s} (w_{id_{ij,s}}, s) - \theta \sum_{k=1}^{N} T_{k,s} (w_{kd_{kj,s}}, s) - \theta S \sum_{s=1}^{S} L_{i,s} = L_i \forall i,$$
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\]

where

\[
\pi_{ij,s} \equiv \frac{X_{ij,s}}{X_{j,s}} = \frac{T_{i,s}(w_{i,d_{ij,s}})^{-\theta}}{\sum_{k=1}^{N} T_{k,s}(w_{k,d_{kj,s}})^{-\theta}};
\]
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\pi_{ij,s} \equiv \frac{X_{ij,s}}{X_{j,s}} = \frac{T_{i,s} (w_i d_{ij,s})^{-\theta}}{\sum_{k=1}^{N} T_{k,s} (w_k d_{kj,s})^{-\theta}};
\]

\[
\sum_{s=1}^{S} L_{i,s} = L_i \forall i,
\]
E V O L U T I O N  O F  T E C H N O L O G Y  P A R T:
LEARNING-BY-DOING AND SPILLOVERS

▷ Labor $L_{i,q}$ generates a mass $\phi L_{i,q}$ of new technologies in $i, q$, share $p_{qs} \in [0, 1]$ of which can be used for any $\omega$ in $i, s$.
EVOLUTION OF TECHNOLOGY PART:
LEARNING-BY-DOING AND SPILLOVERS

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- Over $d\tau$ a mass $dT_{i,s}(t)$ of new technologies arrives to $i$, $s$

\[
    dT_{i,s}(t) = \left( \sum_{q=1}^{s} \phi p_{qs} L_{i,q}(t) \right) d\tau.
\]
EVOLUTION OF TECHNOLOGY PART: LEARNING-BY-DOING AND SPILLOVERS

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- Over $d\tau$ a mass $dT_{i,s}(t)$ of new technologies arrives to $i, s$

$$dT_{i,s}(t) = \left( \sum_{q=1}^{s} \phi p_{qs} L_{i,q}(t) \right) d\tau.$$ 

- Dynamics of sector productivity:

$$T_{i,s}(t) = \int_{\tau=0}^{t} \sum_{q=1}^{s} \phi p_{qs} L_{i,q}(\tau) d\tau + T_{i,s}(0).$$
BALANCED GROWTH PATH (BGP): DEFINITIONS

Dynamics of the model:

\[ \{T_{i,s}\} \rightarrow \{L_{i,s}\} \rightarrow \{T_{i,s} + dT_{i,s}\} \rightarrow \ldots \]
Balanced growth path (BGP): definitions

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Definition: Balanced growth path (BGP) is a sequence of static equilibria along which the state variables \( \{T_{i,s}\}_{i=1,...,N} \) and \( \{L_{i}\}_{i=1,...,N} \) grow at constant rates.
BALANCED GROWTH PATH (BGP): DEFINITIONS

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Definition: Balanced growth path (BGP) is a sequence of static equilibria along which the state variables \(\{T_{i,s}\}_{i \in 1, \ldots, N} \) and \(\{L_{i}\}_{i \in 1, \ldots, N}\) grow at constant rates.

In BGP the relative productivity of sectors remains constant:

\[
\frac{T_{i,s}}{T_{j,s}} = \frac{T_{i,s} + dT_{i,s}}{T_{j,s} + dT_{j,s}} \Rightarrow \frac{T_{i,s}}{T_{j,s}} = \frac{dT_{i,s}}{dT_{j,s}} = \frac{\sum q p_{qs} L_{i,q}}{\sum q p_{qs} L_{j,q}} \quad \forall i, j, s
\]
BGP IN AUTARKY

Let’s start with a simple case of autarky: \( d_{ij,s} \rightarrow \infty \ \forall \ i \neq j \).

- Labor allocation remains the same in each period:
  \[
  L_{i,s} = \alpha_s L_i \quad \text{(follows from } L_{i,s} w_i = \sum_{j=1}^{N} \pi_{ij,s} \alpha_s w_j L_j) \text{, thus, BGP is unique.}
  \]
Now let’s consider another extreme case of zero trade costs: $d_{ij,s} = 1 \forall i, j, s.$
BGP UNDER COSTLESS TRADE

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**Definition:** Matrix of spillovers \( \{p_{qs}\} \) has **no stagnant sectors** if \( \sum_q p_{qs} > 0 \ \forall s. \)
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**Definition:** Balanced growth path is **interior** if \( L_{i,s} > 0 \forall i, s \) and **corner** if at least one \( L_{i,s} = 0. \)
BGP UNDER COSTLESS TRADE

Now let’s consider another extreme case of zero trade costs: $d_{ij,s} = 1 \forall i, j, s$.

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**Definition:** Balanced growth path is **interior** if $L_{i,s} > 0 \forall i, s$ and **corner** if at least one $L_{i,s} = 0$.

**Definition:** Matrix of spillovers $\{p_{qs}\}$ has **no isolated clusters** if its digraph is connected.
BGP AND THE PATTERN OF SPILLOVERS

Without isolated clusters

With isolated clusters

\[
\begin{array}{ccc}
A & B & C \\
A & x & x \\
B & x & \\
C & & \\
\end{array}
\]

\[
\begin{array}{ccc}
A & B & C \\
A & x & x \\
B & x & x \\
C & x & \\
\end{array}
\]
PROPOSITION ON THE UNIQUENESS OF BGP

Proposition: Under zero trade costs, no isolated clusters and no stagnant sectors the model has a unique and stable interior balanced growth path in which labor allocation vectors are the same across countries: \( L_{i,s} = \alpha_s L_i \forall i, s. \)
ILLUSTRATING THE PROPOSITION: PHASE-DIAGRAM

\[ t_B \equiv \frac{T_{j,B}}{T_{i,B}} \]

\[ t_A \equiv \frac{T_{j,A}}{T_{i,A}} \]
ILLUSTRATING THE PROPOSITION: PHASE-DIAGRAM

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\[ t_A \equiv \frac{T_{j,A}}{T_{i,A}} \]

\[ \frac{T_{j,B}}{T_{i,B}} > \frac{T_{j,A}}{T_{i,A}} \]

\[ \frac{T_{j,B}}{T_{i,B}} < \frac{T_{j,A}}{T_{i,A}} \]
ILLUSTRATING THE PROPPOSITION: PHASE-DIAGRAM

\[ t_B \equiv \frac{T_{j,B}}{T_{i,B}} \]

\[ t_A \equiv \frac{T_{j,A}}{T_{i,A}} = \frac{p_{AA}L_{j,A} + p_{BA}L_{j,B}}{p_{AA}L_{i,A} + p_{BA}L_{i,B}} \]

\[ p_{AA} > p_{BA} \]

\[ p_{BB} > p_{AB} \]

\[ \dot{t}_A = 0 \]

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\[ t_A \equiv \frac{T_{j,A}}{T_{i,A}} \]
ILLUSTRATING THE PROPOSITION: PHASE-DIAGRAM

\[
t_B \equiv \frac{T_{j,B}}{T_{i,B}} \quad \text{or} \quad t_B \equiv \frac{T_{j,B}}{T_{i,B}} = \frac{p_{AB}L_{j,A} + p_{BB}L_{j,B}}{p_{AB}L_{i,A} + p_{BB}L_{i,B}}
\]

\[p_{AA} > p_{BA}\]
\[p_{BB} > p_{AB}\]

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ILLUSTRATING THE PROPOSITION: PHASE-DIAGRAM

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ILLUSTRATING THE PROPOSITION: MULTIPLICITY

Isolated sectors: $p_{AA} > p_{BA} = 0$, $p_{BB} > p_{AB} = 0$

\[ t_B \equiv \frac{T_{j,B}}{T_{i,B}} \]
\[ t_A = 0 \]

\[ t_A \equiv \frac{T_{j,A}}{T_{i,A}} = \frac{L_{j,A}}{L_{i,A}} \]
\[ t_B \equiv \frac{T_{j,B}}{T_{i,B}} = \frac{L_{j,B}}{L_{i,B}} \]
ILLUSTRATING THE PROPOSITION: MULTIPLICITY

Isolated sectors: $p_{AA} > p_{BA} = 0$, $p_{BB} > p_{AB} = 0$
ILLUSTRATING THE PROPOSITION: MULTIPLICITY

Isolated sectors: $p_{AA} > p_{BA} = 0$, $p_{BB} > p_{AB} = 0$

$D: t_A = 2 = 1/t_B$

$t_B = 0$

$t_A = 0$

$t_A \equiv \frac{T_{j,A}}{T_{i,A}}$
ILLUSTRATING THE PROPOSITION: UNIQUENESS

Positive inter-sector spillovers: \( p_{AA} > p_{AB} > 0, \ p_{BB} > p_{BA} > 0 \)
ILLUSTRATING THE PROPOSITION: UNIQUENESS

Positive inter-sector spillovers: $p_{AA} > p_{AB} > 0$, $p_{BB} > p_{BA} > 0$
Why no complete specialization?

- Unproductive sectors do not collapse because as they shrink it also becomes relatively easier to improve their productivity: productivity is proportional to $T^{1/\theta}$.
BGP UNDER COSTLESS TRADE: INTUITION

Why no complete specialization?
- Unproductive sectors do not collapse because as they shrink it also becomes relatively easier to improve their productivity: productivity is proportional to $T^{1/\theta}$.

Why inter-sector spillovers eliminate multiplicity of BGPs?
- Less productive sectors gain disproportionally more from inter-sector spillovers than more productive ones.
SUBOPTIMALITY OF BGP

- Competitive BGP can be a suboptimal equilibrium: labor allocation $L_{i,s} = \alpha_s L_i \forall i, s$ doesn’t depend on $\{p_{qs}\}_{q,s\in1,\ldots,S}$. ...
SUBOPTIMALITY OF BGP

- Competitive BGP can be a suboptimal equilibrium: labor allocation \( L_{i,s} = \alpha_s L_i \forall i, s \) doesn’t depend on \( \{p_{qs}\}_{q,s}\in1,...,S \)...

- ...while the condition for welfare-optimal labor allocation, for instance, in autarky is

\[
\frac{1}{L_r} \left( \alpha_r + \frac{1}{\theta} \sum_{s \in S} \frac{\alpha_s L_r p_{rs}}{\sum_{q \in S} L_q p_{qs}} \right) = \frac{1}{L_v} \left( \alpha_v + \frac{1}{\theta} \sum_{s \in S} \frac{\alpha_s L_v p_{vs}}{\sum_{q \in S} L_q p_{qs}} \right)
\]
TOOL OF INDUSTRIAL POLICY

Policy tool: sector-specific taxes \( \{ \tau_{i,s} \} \) that affect the unit cost of production,

\[
\frac{\tau_{i,s} w_i}{z_{i,s}(\omega)}.
\]

Revenues are distributed as a lump-sum transfer to households.
TOOL OF INDUSTRIAL POLICY

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\]

Revenues are distributed as a lump-sum transfer to households.

Same effect on the relative costs as an exogenous sector productivity shifter \( A_{i,s} \) (e.g. climate, deposits of natural resources, etc.):

\[
\frac{w_i}{A_{i,s} z_{i,s}(\omega)}.
\]
INDUSTRIAL POLICY IN AUTARKY

- Optimal taxes – maximizing utility per capita on the BGP. Optimality criteria:

\[
\frac{1}{L_r} \left( \alpha_r + \frac{1}{\theta} \sum_{s \in S} \frac{\alpha_s L_r p_{rs}}{\sum_{q \in S} L_q p_{qs}} \right) = \frac{1}{L_v} \left( \alpha_v + \frac{1}{\theta} \sum_{s \in S} \frac{\alpha_s L_v p_{vs}}{\sum_{q \in S} L_q p_{qs}} \right)
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\]

- For a simple 2 × 2 case under \( \alpha_A = \alpha_B \), sector B should be taxed more than A if \( p_{AA} p_{AB} > p_{BB} p_{BA} \)
INDUSTRIAL POLICY IN AUTARKY: EXAMPLE

Optimal tax $\tau_i^B$ under spillovers

$p_{AA} = p_{BB} = 0.9$, $p_{AB} = 0.7$, $p_{BA} = 0.1$ and

$\alpha_A = \alpha_B = 0.5$, $\theta = 8$;
INDUSTRIAL POLICY IN AUTARKY: SUMMARY

- Subsidize the “core” sectors.
INDUSTRIAL POLICY IN AUTARKY: SUMMARY

▶ Subsidize the “core” sectors.

▶ Optimal policy in autarky depends on the presence of both intra- and inter-sector spillovers.
INDUSTRIAL POLICY IN AUTARKY: SUMMARY

- Subsidize the "core" sectors.

- Optimal policy in autarky depends on the presence of both intra- and inter-sector spillovers.

- If inter-sector spillovers are zero, the optimal tax is zero. The model is similar to the one with equal Marshallian externalities across sectors:

\[ C_{i,s} = L_{i,s} (T_{i,s})^{1/\theta} = L_{i,s}^{1+1/\theta} \left( \phi p_{ss}^s / g \right)^{1/\theta} \]
INDUSTRIAL POLICY IN OPEN ECONOMY: IMPACT ON COMPARATIVE ADVANTAGE

2 × 2 example with positive cross-sector spillovers:

\[ t_B \equiv \frac{T_{j,B}}{T_{i,B}} \]
\[ \tau_{j,A} = 1 = \tau_{j,B} = \tau_{i,A} = \tau_{i,B} \]
\[ t_A = 0 \]
\[ t_B = 0 \]
\[ t_A \equiv \frac{T_{j,A}}{T_{i,A}} \]
INDUSTRIAL POLICY IN OPEN ECONOMY: IMPACT ON COMPARATIVE ADVANTAGE

$2 \times 2$ example with positive cross-sector spillovers:

$t_B \equiv \frac{T_{j,B}}{T_{i,B}}$

$t_A = 0$

$t_B = 0$

$\tau_{j,A} > 1 = \tau_{j,B} = \tau_{i,A} = \tau_{i,B}$

$t_A \equiv \frac{T_{j,A}}{T_{i,A}}$
INDUSTRIAL POLICY IN OPEN ECONOMY: WELFARE

2 × 2 example: \( p_{AA} = p_{BB} = 0.9, \ p_{AB} = 0.7, \ p_{BA} = 0.1 \)
INDUSTRIAL POLICY IN OPEN ECONOMY: WELFARE

2 × 2 example: $p_{AA} = p_{BB} = 0.9$, $p_{AB} = 0.7$, $p_{BA} = 0.1$

$U_i + U_j$
Exogenous comparative advantage in a “non-core” sector can result in losses from trade. Example: symmetric $2 \times 2$, $p_{AA} = p_{BB} = 0.9$, $p_{AB} = 0.7$, $p_{BA} = 0.1$. 

![Graph showing the relationship between $1/A_{i,B}$ and $U_i/L_i$ with two lines representing Autarky and Costless trade. The graph illustrates the impact of trade on productivity, with the Autarky line showing lower productivity compared to Costless trade at lower values of $1/A_{i,B}$.](image-url)
Industrial policy in open economy: summary

- Conditional on no response from trade partners, the optimal labor re-allocation is larger than in autarky, yet, can be attained with lower taxes.
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INDUSTRIAL POLICY IN OPEN ECONOMY: SUMMARY

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- Possibility for welfare losses from trade openness.
Matrix of probabilities of spillovers \( \{p_{qs}\} \) – a new and crucial set of parameters.

Calibration of technological spillovers
Calibration of technological spillovers

- Matrix of probabilities of spillovers \( \{p_{qs}\} \) – a new and crucial set of parameters.

- Using the data on patent citations to quantify \( \{p_{qs}\} \): US patent data 1976-2006 and Japanese 1985-2010 as a robustness check.
CALIBRATION OF TECHNOLOGICAL SPILLOVERS: COHORT APPROACH

Ideas: $p_{qs}$

Origin cohort: $Q(q, t)$

Destination cohort: $Q(s, t + \Delta t)$

Citations: $C(qs, t, t + \Delta t) \sim \text{Poisson} \left( p_{qs} Q(q, t) Q(s, t + \Delta t) \right)$

$$p_{qs}^{MLE} = \frac{\sum_t C(qs, t, t+\Delta t)}{\sum_t Q(q, t) Q(r, t+\Delta t)}$$
CALIBRATION OF TECHNOLOGICAL SPILLOVERS: COHORT APPROACH

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Citations: $C(qs,t,t + \Delta t) \sim Poisson(p_{qs}Q(q,t)Q(s, t + \Delta t))$

$p_{qs}^{MLE} = \frac{\sum_t C(qs,t,t+\Delta t)}{\sum_t Q(q,t)Q(r,t+\Delta t)}$

Split patents for each pair $(q, s)$ into non-overlapping cohorts:

- cited: patents issued within 1 calendar year
- citing: patents issued 2-11 years after the cited cohort
CALIBRATION OF TECHNOLOGICAL SPILLOVERS: SEQUENCE APPROACH

**Patent** $i$ in $q$  

**Patents** in $s$, $K$ of which cite $i$  

$Pr(C(qs) = K) = C^K_N (p_{qs})^K (1 - p_{qs})^{N-K}$

Citations: $C(qs) \sim B(N, p_{qs})$

$p_{qs}^{MLE} = \frac{\sum_{i \in q} K_i(s)}{\sum_{i \in q} N_i(s)}$
CALIBRATION OF TECHNOLOGICAL SPILLOVERS: SEQUENCE APPROACH

- Arrange patents by patent numbers into a sequence
- For each patent from origin sector $q$ find $N$ and $K$ for each destination sector $s$. 

\[ Pr(C(qs) = K) = C_N^K (p_{qs})^K (1 - p_{qs})^{N-K} \]

Citations: 
\[ C(qs) \sim B(N, p_{qs}) \]

\[ p_{qs}^{MLE} = \frac{\sum_{i \in q} K_i(s)}{\sum_{i \in q} N_i(s)} \]
Cohort and sequence approaches for the US
**Intensity of patenting, $\phi$**

- In the model I have $\phi p^q_s$ ideas are generated by unit mass of labor in sector $q$ for sector $s$.

- In reality it is more like $\overline{\phi} \phi_q p^q_s$.

- The equivalent of $\phi$ in the model is $\overline{\phi} = \max_r [\phi_r]$ and the equivalent of $p^q_s$ in the model is $\frac{\phi_q}{\overline{\phi}} p^q_s$.

- This normalization is innocuous since what matters for policy is the relative values of $\{p^q_s\}$ within the matrix.
**P-matrix for the US: logs of spillover probabilities**

\[ \log(p^{qs}) \]

![Heatmap showing the logarithms of spillover probabilities](image-url)
P-matrix for Japan: logs of spillover probabilities

$log(p^{qs})$

-16
-14
-12

$q$

$s$

5 10 15 20

5 10 15 20

5 10 15 20
Spillover probabilities estimates for the US and Japan

\[ \log(p_{qs}^{\text{US}}) \]

\[ \log(p_{qs}^{\text{JP}}) \]
Optimal industrial policy, autarky

Table: Actual ($l^q$), autarky-optimal BGP ($l^q^*$), free market BGP ($\alpha^q$) labor allocations.

<table>
<thead>
<tr>
<th>Sector</th>
<th>$\alpha^q$</th>
<th>$l^q^*$</th>
<th>$l^q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>15.8%</td>
<td>14.4%</td>
<td>27.1%</td>
</tr>
<tr>
<td>Wood products</td>
<td>1.6%</td>
<td>1.4%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Printing and related activities</td>
<td>1.4%</td>
<td>1.3%</td>
<td>2.8%</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Chemical manufacturing</td>
<td>7.6%</td>
<td>8.8%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Computer and electronic products</td>
<td>5.2%</td>
<td>6.3%</td>
<td>5.9%</td>
</tr>
<tr>
<td>Medical equipment and supplies</td>
<td>0.8%</td>
<td>0.9%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>

- Gains in productivity of 3.5% due to moving from the current labor allocation to the optimal one.
- Policy in open economy – still work in progress.
CONCLUDING REMARKS

1. Suggest a framework for analyzing dynamics of comparative advantage.
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2. Prove the importance of inter-sector spillovers for the growth path of comparative advantage.

3. Demonstrate the potential of industrial policy and possibility for losses from trade.

4. Establish the criteria for optimal industrial policy based on strength of technological spillovers.