Retailing with 3D Printing

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Given the promise of 3D printing, also known as additive manufacturing, some innovative consumer goods companies have started to experiment with such a technology for on-demand production. However, the potential impact of 3D printing on retail and supply chain operations is not well understood. In this paper, we consider two adoption cases of 3D printing in a dual-channel (i.e., online and in-store) retail setting, and evaluate its impact on a firm’s product offering, prices for the two channels, as well as inventory decisions. Our analysis uncovers the following effects of 3D printing. First, 3D printing at the factory has the substitution effect of technological innovation for online demands, as 3D printing replaces the traditional mode of production. Such technology substitution not only leads to increased product variety offered online, which allows the firm to charge a price premium for online customers, but also induces the firm to offer a smaller product variety and a reduced price in-store. Second, when 3D printing is used in-store as well, in additional to the substitution effect, the firm also achieves a structural effect due to the fundamental change in the supply chain structure. Since the in-store demand is served in a build to order fashion, the firm achieves postponement benefits in inventory management. Moreover, using 3D printing in-store will require a new supplier-retailer relationship. We find that cost-sharing contracts can coordinate the supply chains where 3D printing is used in-store and the supplier controls the raw material inventory.

Key words: 3D printing; build to order; dual channels; product variety; pricing; supply chain management

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1. Introduction

3D printing as a manufacturing process has come a long way. It was first introduced in the late 1980s as a rapid prototyping technology (3D Printing Industry 2016). Also known as additive manufacturing, the manufacturing process involves creating a three dimensional object by adding layers of materials, such as molten polymers, metal powders, or even bio-tissues, until the final shape is complete. Since it does not require the construction of injection moulding for mass-production processes, 3D printing has the promise that you can build precise, customized products without the required large batch sizes to justify the setup. As such, manufacturers have experimented with using the technology beyond prototyping, but for real production needs, such as Adidas and GE (Economist 2017). The promises of 3D printing to production are many – companies can now offer
much more product variety than before, manufacturing can be moved from the traditional build-to-
stock (BTS) mode to build-to-order (BTO) mode, manufacturing can be distributed from factories
to retailers or even to consumers, and improved customer satisfaction from perfect fit products
and fast response times for tailored-made products. This paper aims at analytically understanding
some of these benefits of 3D printing.

Of course, 3D printing may not be for every product. Roca et al. (2017) cautioned the “hype”
about 3D printing. The often-cited example of GE has been about the company’s investment on
a factory to print fuel nozzles for the new LEAP jet engine (Economist 2017). Industrial goods
(such as parts for jet engine) are complex, with large demand uncertainty due to low volume.
Such complexity means that 3D printing is more suited for fast prototyping and/or small-scale
production for industrial goods (Roca et al. 2017, Song and Zhang 2016). On the other hand,
consumer goods (such as wearables and jewelry) are much simpler to produce, with relatively
smaller demand uncertainty due to high volume (Roca et al. 2017). Hence, 3D printing is better
positioned to enable localized production and mass customization for consumer goods. For this
reason, we focus our analysis on the potential impact of 3D printing on consumer goods retailing.

To understand the impact of 3D printing on consumer goods, we have to recognize that the
consumer goods supply chain has also undergone significant evolution due to technological advances
in e-commerce. Online sales have been increasing worldwide at unprecedented rates (e.g., it was
19.4% at Amazon, 40% at Apple, 9% at Walmart, and 12.5% at Macy’s; see Zaczkiewicz 2017).
Hence, it would not be appropriate to analyze 3D printing in the absence of e-commerce. It is
also interesting to note that, among the top 10 US retailers with the highest e-commerce sales in
2017, eight of them also sell through their own stores. Dual channels (with sales via both brick-
and-mortar stores and online) have become a mainstream means for companies to interact with
consumers.

Dual channels thus form the basis of our analysis of 3D printing. The advance of internet tech-
nologies, in the absence of 3D printing, has established the combination of online and in-store
channels as the mainstream sales channel for brands and merchants. The emergence of 3D printing
will likely be first introduced to factories of such merchants (such as the Adidas example cited
by Economist 2017). The high cost of 3D printers (e.g., HP entered the market in 2016 with 3D
plastic printers costing from $130,000 up) could also be a reason why the adoption of 3D printing
would start with the factory. In this case, the factory would serve two kinds of demands: online
consumer demand and store replenishment. In the latter, since store replenishment is still in bulk,
the factory can continue to use mass production systems for cost efficiency. 3D printing, however,
could suit the needs of online consumers. As 3D printing technologies continue to diffuse, and lower
cost printers become available, it is then feasible economically for the stores to adopt 3D printing. When that happens, mass customization can also occur at the stores.

The three cases of interest are thus as follows: 1) the base case in which the firm offers a finite number of product options, and uses BTS (build-to-stock) to satisfy in-store demands while BTO (build-to-order) for the online demands; 2) 3D printing is adopted at the factory so that it can use BTO and mass customization to satisfy online orders, while keeping BTS with a finite number of product options for in-store demands; and 3) 3D printing is also adopted at the stores, so that both online and in-store demands are satisfied under the BTO mode with mass customization options. This hypothetical diffusion path is illustrated in Figure 1.\(^1\) Comparing Case 2 to Case 1 shows the effect of technology substitution, where one form of manufacturing was replaced by another. Comparing Case 3 to Case 2 shows the effect of structural change of the supply chain, where mass customization as a production mode is carried out at retail store level.

\(^1\) Note that the empty cell in the two-by-two matrix is an unlikely scenario in which the firm deploys 3D printing at stores while still keeping traditional BTO manufacturing at the factory. We omit this case in our analysis.
when these two entities may belong to two organizations. Specifically, if the two are independent organizations, how can the supply chain be coordinated by incentive and contract design in the case of 3D printing in-store?

We develop a stylized model to address the above questions. Even though the model is stylized in nature, we have included in it features that capture as much as possible the underlying dynamics and trade-offs faced by merchants and consumers. In our model, consumers as customers are heterogeneous in two dimensions: 1) the waiting cost they incur from purchasing online, and 2) the fit cost they incur because the product types offered by the merchant may not meet their needs exactly. We formulate an integrated optimization problem involving the following decisions: number of horizontally differentiated products to offer, prices for online and in-store channels, as well as the corresponding inventory decisions. We will conduct the analysis for two key types of products, following the distinction of Fisher (1997) – functional products with low demand uncertainties, and innovative products with high demand uncertainties.

We highlight a few key insights from the analysis here. First, as shown in Figure 1, the traditional system employs BTO for the online channel and BTS for the in-store channel. As a result, the inventory mismatching cost for the in-store channel is much higher than that of the online channel. For innovative products with high demand uncertainty, we show that this production mode difference would induce the optimal online price to be lower than the in-store price, so that the firm can attract more demand from the in-store channel to the online channel to take advantage of the BTO production process. Moreover, due to the high in-store price, a proportion of customers would be left unserved at the equilibrium.

Second, adopting 3D printing for online customers gives rise to a substitution effect of technological innovation, i.e., the traditional production technology is replaced by a better 3D printing technology (Lee 2007). Such technology substitution leads to the variety effect, enabled by 3D printing’s natural elimination of the production setup cost, and allows the firm to offer perfect customization and charge a price premium for online customers. However, this “higher online price” effect has to be balanced against the opposite “lower online price” effect induced by the BTO/BTS difference of the online and in-store channels (as discussed above). Specifically, we show that for functional products with low demand uncertainty, the firm should charge a price premium for the 3D-printed products. However, for innovative products with high demand uncertainty, the firm should still set the online price lower than the in-store price to attract more demand to the online channel. Thus, different pricing strategies are needed for different product characteristics in this case. Moreover, under certain scenarios, the firm may go to the extreme to completely shut off the in-store channel, so as to eliminate the in-store inventory mismatching cost. Compared to the traditional system, we show that adopting 3D printing online will reduce the variety of in-store product
offering. However, when both channels are in use, unlike the traditional system, no customers will be left unserved regardless of the product demand uncertainty. The demand segmentation between the online and in-store channels depends on the customers’ online purchase waiting cost.

Third, when 3D printing is used at stores, the substitution effect is also compounded by the structural effect (Lee 2007) due to the change in the supply chain structure. Manufacturing now occurs at the store as well. With 3D printing in-store, the in-store demands are now served by BTO instead of BTS, which naturally leads to the postponement effect. This effect allows the firm to achieve both inventory pooling and overage cost reduction. Moreover, with perfect customization in both channels, the firm can charge a full price premium for the in-store channel. The optimal price for the online channel is either a full price premium or a discounted one, depending on the online waiting cost and product demand uncertainty. Because the symmetry of the degree of product customization is restored in the two channels, the demand segmentation reverses to one similar to that under the traditional system (where the degree of product customization is also symmetric between the two channels). However, in this case, unlike the traditional system, no customers will be left unserved regardless of the product demand uncertainty.

Finally, another important implication of supply chain structural change from 3D printing in-store is that it will require a new supplier-retailer relationship, if the stores belong to a different organization from the firm. We show that cost-sharing contracts can coordinate the supply chain of 3D printing in-store where the supplier controls the raw material inventory. This new form of contracts complements the well-known supply-chain-coordinating buy-back and revenue-sharing contracts in the literature. We project that, with the emerging trend of on-demand production such as 3D printing in-store, cost-sharing contracts between suppliers and retailers will start to gain traction in practice.

The remainder of this paper is organized as follows. After a review of relevant literature in §2, we describe in §3 the model setup for the three cases under consideration. In §4, we analyze the three cases and derive analytical insights about the impact of 3D printing. We validate and strengthen our insights numerically in §5. We conclude the paper in §6 with discussions of managerial implications and future research directions.

2. Literature Review

There is an emerging stream of research that studies the impact of 3D printing technology on manufacturing and supply chain management. For example, Song and Zhang (2016) develop a queueing model to analyze and quantify the impact of 3D printing on spare parts logistics. In this application, 3D printing technology is used for small-scale on-demand production of certain industrial goods spare parts to reduce supply lead time and inventory cost. As discussed earlier,
the impact of 3D printing on industrial goods and consumer goods are different. Our paper focuses on the impact of 3D printing on consumer goods retailing.

Dong et al. (2016) study the impact of 3D printing on a firm’s manufacturing strategy and product assortment decision. They consider three types of manufacturing technology: dedicated technology, traditional flexible technology, and 3D printing technology. The authors show that, while adopting the traditional flexible technology in addition to the dedicated one may reduce product variety chosen by the firm, 3D printing technology always helps increase product variety when used in combination with the dedicated technology. Our paper differs from their paper in several aspects. First, we consider a dual-channel retail setting where the firm sells through both online and in-store channels. Second, we incorporate the firm’s pricing decisions, and study how the product prices should be different under traditional manufacturing technology and 3D printing technology as well as across channels. In our model, demand is endogenously determined by the firm’s product offering and pricing decisions as well as consumers’ heterogeneous preferences regarding product fit and online purchase waiting cost. Finally, we consider a forward-looking case in which 3D printing technology completely replaces the traditional high-setup-cost dedicated technology in one or both channels.

We model customers’ heterogeneous product preferences using the circular city framework, which is a variant of the classic Hotelling (1929) model. This modeling framework has been commonly used to study product differentiation and product line design over horizontally differentiated products (e.g., Salop 1979, Riordan 1986, Dewan et al. 2003). A novel aspect of our work is the focus on the integrated analysis of product offering, pricing and inventory operations. This aspect is different from the traditional product line design literature where the firm makes the profit-maximizing product offering and pricing decisions by ignoring demand uncertainty and thus the inventory operations costs, e.g., Mussa and Rosen (1978), Moorthy (1984), and Desai (2001). It is worth commenting that Netessine and Taylor (2007) study the effect of economic order quantity (EOQ) inventory costs in a product line design problem with deterministic demand; our model complements theirs by concerning the effect of newsvendor inventory costs under stochastic demand. Moreover, we study the problem in a dual-channel setting where the firm can offer different degrees of product customization between the online and in-store channels. These new features of our setting enables us to gain deeper insights regarding the interactions between the (induced) online and in-store demands when 3D printing technology is adopted online and/or in-store.

Our dual-channel model is also conceptually related to work of Gao and Su (2017a,b) who study various problems of omni-channel retailing in the presence of strategic consumers, such as the effects of buy-online-and-pick-up-in-store, physical showroom, virtual showroom, and providing real-time store inventory availability information. They model the strategic interaction between the firm and
the consumers as a simultaneous-move game. While our model also involves the online and in-store channels, we assume a sequential-move Stackelberg game framework where the firm moves first to set the product offering and pricing decisions, followed by consumers making their rational choice among the product variants and between the two channels to maximize their own utility.

Our paper is also related to the literature of postponement and mass customization. Lee (1996) studies the product/process postponement design in both BTO and BTS production modes. Lee and Tang (1997) study the optimal point of product differentiation (i.e., the stage after which the products assume their unique identities). Jiang et al. (2006) consider a mass customization system consisting of an initial BTS stage and a final BTO stage. Alptekinoğlu and Corbett (2008) study the competition between a mass customizer that can offer any variety within a product space (similar to the 3D printing technology in our model), and a mass producer that offers a finite set of products in the same space (similar to the traditional technology in our model). Alptekinoğlu and Corbett (2010) further study the trade-off between the increased ability to precisely meet customer preferences and the increased lead time from order placement to delivery associated with customized products. Our model captures a simple, binary version of this trade-off in the 3D printing online case, i.e., with a fixed lead time from the online channel, customer preferences can be met precisely. Our dual-channel model features two production modes in parallel, where each channel can use a different production mode depending on the technology adoption. Moreover, we show that adopting 3D printing (BTO) in-store can lead to significant supply chain structural changes. Specifically, a retail store is turned into a local production site, and, as a result, the firm can achieve additional postponement and inventory pooling benefits.

Finally, our paper also contributes to the literature of supply chain contracting (see Cachon 2003 for a review). In particular, we show that cost-sharing contracts can coordinate the supply chain of 3D printing in-store where the supplier controls the raw material inventory. This new form of contracts makes a nice complement to the existing supply-chain-coordinating buy-back and revenue-sharing contracts (e.g., Wang et al. 2004, Cachon and Lariviere 2005).

3. Model Setup

We consider a firm that produces and sells products in two channels, a brick-and-mortar store channel and an online channel. Hereafter, without specific mention, “in the store” or “in-store” means in the brick-and-mortar store. We assume that in-store demand and online demand are endogenously determined by customers’ channel preferences and product preferences, which are specified as follows. Customers are heterogeneous in two dimensions: 1) the waiting cost they incur from purchasing online, and 2) the fit cost they incur because the product types offered by the firm do not meet their needs exactly.
Specifically, customers’ heterogeneous online waiting costs are captured by the three types of customers described as follows.

**Type I customers (zero online waiting cost).** These customers do not incur waiting cost when purchasing online. We further assume that they choose online when indifferent between purchasing online and in-store. For example, some people are prone to shopping online because they want to avoid traveling to the brick-and-mortar store or fear stockout at the store (Gao and Su 2017a,b). From a survey study, Konuš et al. (2008) find that 37% of respondents tend to use the Internet and catalogs for both information search and purchase. In our model, Type I customers correspond to $\alpha$ proportion of the population ($0 < \alpha < 1$).

**Type II customers (positive online waiting cost).** These customers incur a finite waiting cost $e > 0$ when purchasing online. For example, by purchasing online, customers forgo the joy of receiving the product immediately, which might create a disutility for some people. The existence of customer disutility from purchasing online has been empirically established, such as Bart et al. (2005) and Forman et al. (2009). Type II customers correspond to $\beta$ proportion of the population ($0 < \beta < 1$ and $\alpha + \beta < 1$).

**Type III customers (infinite online waiting cost).** These customers have infinite online waiting cost, so they effectively shop only from the brick-and-mortar store. In the study of Konuš et al. (2008), 23% of respondents are “store-focused” who reveal favorable attitudes toward brick-and-mortar stores. In our model, Type III customers correspond to the remaining $1 - \alpha - \beta$ proportion of the population.

![Figure 2 Illustration of the circular city customer utility model.](image-url)

Besides customers’ heterogeneous channel preferences resulting from the waiting cost differences, we model customers’ heterogeneous product preferences using the circular city framework, which is a variant of the classic Hotelling (1929) model. This modeling framework has been used to study...
product differentiation and product line design over horizontally differentiated products (e.g., Salop 1979, Riordan 1986, Dewan et al. 2003). We assume that customers are located on a circle of unit circumference. Customers are uniformly distributed on the circle. Each customer’s location represents her ideal product type (e.g., her size or favorite color of a product), and the arc distance between a product location and the customer location measures the customer’s misfit from this product. Each customer only purchases the product type that is closest to her location on the circle. Given an in-store price $p$, the customer’s utility from purchasing in-store a product that is $x$ arc distance away is $v - p - tx$, where $v$ is the valuation of customers for the ideal product type, $t$ is the fit cost parameter (which corresponds to the transportation cost parameter in the classic Hotelling model) and measures customers’ sensitivity to product differences, and $tx$ is the fit cost of customer $x$. Similarly, given an online price $p$, the customer’s utility is $v - p - tx - e$ from purchasing online (note that $e = 0$ for Type I customers and $e = \infty$ for Type III customers).

See Figure 2 for an illustration.

The total customer demand from the unit circle follows a normal distribution with mean $\mu$ and standard deviation $\sigma$. We assume that the demand at each point on the customer circle follows an i.i.d. normal distribution. Thus, the demand from a customer segment with arc length $x$ follows a normal distribution with mean $\mu x$ and standard deviation $\sigma \sqrt{x}$. Let $f_o(\cdot)$ and $f_i(\cdot)$ denote the normal probability density function for the online and in-store demands, respectively. Additionally, let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal probability density function (pdf) and cumulative density function (cdf), respectively.

To meet the demand from the two channels, the firm may use the traditional production technology or adopt the 3D printing technology in one or both channels. With 3D printing, the firm can effectively offer “infinite” types of product to cover the entire customer circle and customers do not incur any fit costs. To keep things simple, we assume that producing one unit of product requires one unit of common raw material under both traditional and 3D printing technologies, and that the product quality is the same under both technologies. We consider three cases described as follows and illustrated in Figure 3.

**Case 1: Traditional system.** The firm uses the traditional technology to produce products sold in both channels. Due to the fixed production setup cost for different product types, the traditional system can only entertain a finite number of product offerings. The firm’s production is build-to-stock (BTS) for in-store demand and build-to-order (BTO) for online demand. For in-store demand, the firm distributes finished goods to the brick-and-mortar store and holds inventory in the form of finished goods in the store. For online demand, the firm holds inventory in the form of raw material in the factory (see Case 1 of Figure 3).
Case 2: 3D printing online. The firm uses the 3D printing technology for meeting online demand only. Without 3D printing in-store, the firm can only stock a finite number of product offerings in the store. In this case, the firm uses both types of technology in its production in the factory. The firm’s production is BTS for in-store demand and BTO for online demand. The firm holds finished goods inventory in the store and raw material inventory in the factory for meeting online demand (see Case 2 of Figure 3).

Case 3: 3D printing in-store. The firm uses the 3D printing technology for meeting both online and in-store demands. This involves installing 3D printers both in the factory and in the store. In this case, the firm makes the production for online demand in the factory and makes the production for in-store demand in the store. Both productions are BTO. The firm holds inventory in the form of raw material both in the factory and in the store (see Case 3 of Figure 3).

We assume that the firm incurs marginal cost $c$ for each unit of product, regardless of the product type. To keep things simple, we assume that the marginal cost remains the same when 3D printing is adopted (this is not unreasonable as the costs under 3D printing are expected to drop significantly when the technology reaches its maturity; see Roca et al. 2017). We further decompose the product
cost \( c \) as \( c = c_r + c_p \), where \( c_r \) is the raw material procurement and distribution cost, and \( c_p \) is the production cost. Depending on whether the production mode is BTO or BTS and whether the production is made in the factory or in the store, different components of the marginal cost may be incurred before or after demand realizations (see detailed discussion in §4.1–4.3). Moreover, when using the traditional technology, the firm incurs a production setup cost \( s \) for each product type it offers, due to factors such as making product molds, switchover and/or retooling. Thus, given \( n \) product types, the total setup cost is \( sn \). Additionally, the firm incurs a fixed cost to purchase the 3D printers and train employees for each channel where it adopts the 3D printing technology. This fixed cost is \( k \) at the factory and \( k' \) at the store.

4. Model Analysis: The Impact of 3D Printing

In each of the three cases illustrated in Figure 3, the firm needs to make four decisions: number of horizontally differentiated products to offer,\(^2\) prices for products sold online and in-store, as well as inventory decisions for the in-store channel.\(^3\) In the next three subsections, we analyze the firm’s optimal strategies in each production case. Then, by comparing the optimal strategies across different cases, we obtain how the adoption of 3D printing affects the firm’s product offering, pricing and inventory decisions in each channel, and develop insights regarding how 3D printing creates value to the firm.

4.1. Case 1: Traditional System

In Case 1 (traditional system; see Figure 3), the firm uses the traditional technology to produce \( n \) types of horizontally differentiated products, and chooses price \( p_o \) for all products sold online and \( p_i \) for all products sold in-store (subscript “\( o \)” represents online and subscript “\( i \)” represents in-store). Because product types are horizontally differentiated (i.e., differentiated in a dimension other than quality), the firm charges the same price for all product types within each channel. For example, apparel producers usually charge the same price for all sizes and colors of the same style, however the same item may be sold at a different price in the online store compared to the brick-and-mortar store.

To derive the firm’s optimal strategy, we need to first characterize the customer choices. Consider the arc on the customer circle that is centered at the location of any product type and has arc length \( 1/n \) \( (n \geq 1) \). This arc corresponds to the demand base for this product type. Moreover, the customers’ utilities are symmetric on two sides of the product location. Thus, to analyze the customer choices, we focus on the arc on one side of the product location, where the customer’s

\(^2\) Under Case 3, this decision is eliminated due to the perfect customization enabled by 3D printing in both channels.

\(^3\) Note that no inventory decisions are required for the online channel in our model, as we assume that the factory producing under the BTO mode can source raw material in a just-in-time manner (see §4.1 for a discussion).
distance from her ideal product type, \( x \), ranges in \( 0 \leq x \leq 1/(2n) \). We derive the purchasing decisions of each type of customers as follows.

- **Type I customers**: Their utility from purchasing online is \( v - p_o - tx \), and their utility from purchasing in-store is \( v - p_i - tx \). Then, Type I customers purchase online if \( v - p_o - tx \geq v - p_i - tx \) and \( v - p_o - tx \geq 0 \), purchase in-store if \( v - p_i - tx > v - p_o - tx \) and \( v - p_i - tx \geq 0 \), and do not purchase otherwise. Thus, Type I customers purchase online if \( p_i - p_o \geq 0 \) and \( 0 \leq x \leq \min\left(\frac{v - p_o}{t}, \frac{1}{2n}\right) \), and purchase in-store if \( p_i - p_o < 0 \) and \( 0 \leq x \leq \min\left(\frac{v - p_o}{t}, \frac{1}{2n}\right) \).

- **Type II customers**: Their utility from purchasing online is \( v - p_o - tx - e \), and their utility from purchasing in-store is \( v - p_i - tx \). Then, Type II customers purchase online if \( v - p_o - tx - e \geq v - p_i - tx \) and \( v - p_o - tx - e \geq 0 \), purchase in-store if \( v - p_i - tx > v - p_o - tx - e \) and \( v - p_i - tx \geq 0 \), and do not purchase otherwise. Thus, Type II customers purchase online if \( p_i - p_o \geq e \) and \( 0 \leq x \leq \min\left(\frac{v - p_o - e}{t}, \frac{1}{2n}\right) \), and purchase in-store if \( p_i - p_o < e \) and \( 0 \leq x \leq \min\left(\frac{v - p_o}{t}, \frac{1}{2n}\right) \).

- **Type III customers**: Their utility from purchasing in-store is \( v - p_i - tx \), so Type III customers purchase in-store if \( v - p_i - tx \geq 0 \), and do not purchase otherwise. Thus, Type III customers purchase in-store if \( 0 \leq x \leq \min\left(\frac{v - p_i}{t}, \frac{1}{2n}\right) \).

Based on the customer choices, we obtain that the proportion of customers purchasing online is

\[
d_o(n, p_o, p_i) = \begin{cases} 
\alpha \min \left(\frac{v - p_o}{t}, \frac{1}{2n}\right) + \beta \min \left(\frac{v - p_o - e}{t}, \frac{1}{2n}\right) & \text{if } p_i - p_o > e, \\
\alpha \min \left(\frac{v - p_o}{t}, \frac{1}{2n}\right) & \text{if } 0 \leq p_i - p_o \leq e, \\
0 & \text{if } p_i - p_o < 0,
\end{cases}
\]

and the proportion of customers purchasing each product type in-store is

\[
d_i(n, p_o, p_i) = \begin{cases} 
2(1 - \alpha - \beta) \min \left(\frac{v - p_i}{t}, \frac{1}{2n}\right) & \text{if } p_i - p_o > e, \\
2(1 - \alpha) \min \left(\frac{v - p_i}{t}, \frac{1}{2n}\right) & \text{if } 0 \leq p_i - p_o \leq e, \\
2 \min \left(\frac{v - p_i}{t}, \frac{1}{2n}\right) & \text{if } p_i - p_o < 0.
\end{cases}
\]

Depending on the relationship between \( p_o \) and \( p_i \), the demand segmentation takes different forms and as is shown above, there are three possible scenarios. Without loss of generality, we restrict the prices to \( 0 \leq p_o, p_i \leq v \) in all our analyses, as a price higher than \( v \) does not yield any sales. In addition, consistent with previous literature (e.g., Salop 1979, Riordan 1986, De Groote 1994), we ignore the integer constraint for \( n \).

The total online demand \( D_o \) follows a normal density \( f_o(\cdot) \) with mean \( \mu_o(n, p_o, p_i) = d_o(n, p_o, p_i)\mu \) and standard deviation \( \sigma_o(n, p_o, p_i) = \sigma \sqrt{d_o(n, p_o, p_i)} \). The demand for each product type in the store \( D_i \) follows a normal density \( f_i(\cdot) \) with mean \( \mu_i(n, p_o, p_i) = d_i(n, p_o, p_i)\mu \) and standard deviation \( \sigma_i(n, p_o, p_i) = \sigma \sqrt{d_i(n, p_o, p_i)} \). Note that the total in-store demand across all product types is \( nD_i \).
When the firm uses the traditional system, to meet online demand, the firm makes production in a BTO fashion and holds inventory in the form of raw material in the factory; to meet in-store demand, the firm makes production in a BTS fashion and holds inventory in the form of finished goods in the store. The raw material inventory responds to the realized total demand of all product types offered online. For in-store demand, the firm needs to decide the inventory order quantities for each product type. Recall that the horizontally differentiated products are symmetrical, so the inventory decisions are identical across different product types, which we denote as \( q \).

Therefore, the firm’s total profit can be written as

\[
\Pi(n, p_o, p_i, q) = \Pi_o(n, p_o, p_i) + \Pi_i(n, p_o, p_i, q) - s_n,
\]

where \( \Pi_o(n, p_o, p_i) \) and \( \Pi_i(n, p_o, p_i, q) \) are the profits from the online and in-store channels, respectively, and \( s_n \) is the setup cost for \( n \) product types.

Specifically, the firm’s profit from the online channel is

\[
\Pi_o(n, p_o, p_i) = (p_o - c) d_o(n, p_o, p_i) \mu. \tag{1}
\]

Here we assume that the factory producing for the online channel can source raw material in a just-in-time manner. As a result, there is no inventory mismatching cost for the online channel. There are many industry examples to support this assumption. For example, it is possible to have suppliers co-located with the manufacturer, such as Toyota (Toyota City) in Japan and the Smart Car (Smartville). Some companies (e.g., Dell, Apple, Cisco, Samsung Electronics, Volkswagen) also use supplier hubs (or vendor hubs), so even though the supplier’s factories are not co-located, the supplier has to stock inventory usually very near the manufacturer. Moreover, many companies use the vendor-managed inventory (VMI) scheme with suppliers, so that the suppliers are responsible for stocking and replenishing the inventory at the manufacturer’s site. The suppliers often retain ownership of the inventory (i.e., VMI with consignment), and so the manufacturer can use raw material in a just-in-time manner (Lee and Whang 2008).

The firm’s profit from the in-store channel is

\[
\Pi_i(n, p_o, p_i, q) = \left[ p_i E \left[ \min(D_i, q) \right] - cq \right] n
= \left[ (p_i - c) d_i(n, p_o, p_i) \mu - c \int_{-\infty}^{q} (q - x) f_i(x) \, dx - (p_i - c) \int_{q}^{\infty} (x - q) f_i(x) \, dx \right] n.
\]

The first term of the above expression is the expected profit when there is no demand uncertainty. The last two terms are the newsvendor overage and underage costs, respectively. In this case, production mode is BTS, so the unit overage cost is \( c = c_r + c_p \) and the unit underage cost is \( p_i - c \).
It follows from standard newsvendor analysis that the optimal order quantity is $q^*(n, p_o, p_i) = \mu_i(n, p_o, p_i) + z^*(p_i)\sigma_i(n, p_o, p_i)$ where

$$z^*(p_i) = \Phi^{-1}\left(1 - \frac{c}{p_i}\right)$$

and $\Phi(\cdot)$ is the standard normal cdf. Substituting in $q = q^*(n, p_o, p_i)$ and leveraging the normal distribution property, one can show that the in-store profit function reduces to

$$\Pi_i(n, p_o, p_i) = \left[(p_i - c)d_i(n, p_o, p_i)\mu - p_i\phi(z^*(p_i))\sigma\sqrt{d_i(n, p_o, p_i)}\right]n$$

where $\phi(\cdot)$ is the standard normal pdf and $h(\cdot) = \phi(\cdot)/(1 - \Phi(\cdot))$ is the standard normal hazard function. It is worth noting that the second term in (2) is the expected inventory mismatch (i.e., overage and underage) cost, which is proportional to the finished good inventory unit cost $c$ (because finished good is used as safety stock). Since the normal hazard function $h(\cdot)$ is an increasing function, it is intuitive that the cost term is increasing in the safety stock factor $z^*(p_i)$.

From (1) and (2), the firm’s total profit under the traditional system is

$$\Pi(n, p_o, p_i) = (p_o - c)d_o(n, p_o, p_i)\mu + (p_i - c)d_i(n, p_o, p_i)\mu - ch(z^*(p_i))\sigma_n\sqrt{d_i(n, p_o, p_i)} - sn. \quad (3)$$

Let $n^*, p^*_o, p^*_i$ be the optimal solution that maximizes (3), and $\Pi^*$ be the resulting optimal profit when $n \geq 1$, where we use superscript "*" to denote the optimal decisions and outcomes in the traditional system case. In the following proposition, we characterize structural properties of the optimal strategy in the traditional system.

**Proposition 1.** Under the traditional system, the following hold:

(i) If $\Pi^* \geq 0$, the optimal strategy satisfies $p^*_o \leq p^*_i$, and $p^*_o = v - \frac{t}{2n^*}$ or $p^*_o = v - \frac{t}{2n^*} - e$. The optimal customer coverage is full circle if and only if $p^*_o = p^*_i$.

(ii) If $\Pi^* < 0$, no product is offered in either channel, i.e., $n^* = 0$.

Proposition 1 states that under the traditional system, the firm’s optimal online price should be lower than or equal to the optimal in-store price. Under the traditional system, the firm offers the same degree of customization in both channels. However, the production mode is BTO for the online channel and BTS for the in-store channel. Because BTO eliminates the inventory mismatching cost for the online channel, it is optimal for the firm to offer a lower price in the online channel to attract more demand from the in-store channel to the online channel. This is consistent with the current industry practice of online retailers, as prices are usually lower (or the same) in online stores. Proposition 1 also shows that under the traditional system, the optimal product offering
is associated with the online price. Moreover, when the two prices diverge, with the in-store price becoming strictly higher, a proportion of the in-store customers will be left unserved.

To gain further insights, we analyze the special scenario with \( \sigma = 0 \) where we can obtain closed-form solutions. A detailed analysis of the scenario can be found in Appendix B. Using the results from this scenario, we can derive analytical comparison results between the traditional system case and the other cases in the following subsections for sufficiently small \( \sigma \).

### 4.2. Case 2: 3D Printing Online

In Case 2 (3D printing online; see Figure 3), the firm uses the traditional technology to produce \( n \) types of products sold in-store. For online demand, because of the 3D printing technology, the firm can customize products according to each customer’s need and customers do not incur any misfit. Thus, the firm effectively offers infinite product types online.

Same as in Case 1, in order to characterize the customer choices, we consider the arc on one side of any product location with customer distance ranging in \( 0 \leq x \leq 1/(2n) \). The analysis is as follows.

- **Type I customers:** Their utility from purchasing online is \( v - p_o \), and their utility from purchasing in-store is \( v - p_i - tx \). Then, Type I customers purchase online if \( v - p_o \geq v - p_i - tx \) and \( v - p_o \geq 0 \), purchase in-store if \( v - p_i - tx > v - p_o \) and \( v - p_i - tx \geq 0 \), and do not purchase otherwise. Thus, Type I customers purchase online if \( \min(\frac{p_o - p_i}{t}, \frac{1}{2n}) \leq x \leq \frac{1}{2n} \), and purchase in-store if \( 0 \leq x < \min(\frac{p_o - p_i}{t}, \frac{1}{2n}) \). Note that if \( p_i - p_o \geq 0 \), all Type I customers purchase online.

- **Type II customers:** Their utility from purchasing online is \( v - p_o - e \), and their utility from purchasing in-store is \( v - p_i - tx \). Then, Type II customers purchase online if \( v - p_o - e \geq v - p_i - tx \) and \( v - p_o - e \geq 0 \), purchase in-store if \( v - p_i - tx > v - p_o - e \) and \( v - p_i - tx \geq 0 \), and do not purchase otherwise. Thus, Type II customers purchase online if \( p_o \leq v - e \) and \( \min(\frac{p_o - p_i + e}{t}, \frac{1}{2n}) \leq x \leq \frac{1}{2n} \), and purchase in-store if \( 0 \leq x < \min(\frac{p_o - p_i + e}{t}, \frac{1}{2n}) \).

- **Type III customers:** Same as in Case 1, Type III customers purchase in-store if \( 0 \leq x \leq \min(\frac{p_o - p_i + e}{t}, \frac{1}{2n}) \).

Based on the customer choices, we obtain that the proportion of customers purchasing online is

\[
d_o(n, p_o, p_i) = \begin{cases} 
\alpha \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right)^+ 2n & \text{if } p_i - p_o < 0 \text{ and } p_o \geq v - e, \\
\alpha \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right)^+ + \beta \left( \frac{1}{2n} - \frac{p_o - p_i + e}{t} \right)^+ 2n & \text{if } p_i - p_o < 0 \text{ and } p_o < v - e, \\
\alpha + \beta \left( \frac{1}{2n} - \frac{p_o - p_i + e}{t} \right)^+ 2n & \text{if } p_i - p_o \geq 0 \text{ and } p_o \geq v - e, \\
\alpha + \beta \left( \frac{1}{2n} - \frac{p_o - p_i + e}{t} \right)^+ 2n & \text{if } 0 \leq p_i - p_o \leq e \text{ and } p_o < v - e, \\
\alpha + \beta \left( \frac{1}{2n} - \frac{p_o - p_i + e}{t} \right)^+ 2n & \text{if } p_i - p_o > e \text{ and } p_o < v - e,
\end{cases}
\]
and the proportion of customers purchasing each product type in-store is

\[
d_i(n, p_o, p_i) = \begin{cases} 
2 \left[ \alpha \min \left( \frac{p_o - p_i}{t}, \frac{1}{2n} \right) + (1 - \alpha) \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) \right] & \text{if } p_i - p_o < 0 \text{ and } p_o \geq v - e, \\
2 \left[ \alpha \min \left( \frac{p_o - p_i}{t}, \frac{1}{2n} \right) + \beta \min \left( \frac{p_o - p_i + e}{t}, \frac{1}{2n} \right) + (1 - \alpha - \beta) \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) \right] & \text{if } p_i - p_o < 0 \text{ and } p_o < v - e, \\
2(1 - \alpha) \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) & \text{if } p_i - p_o \geq 0 \text{ and } p_o \geq v - e, \\
2 \beta \min \left( \frac{p_o - p_i + e}{t}, \frac{1}{2n} \right) + (1 - \alpha - \beta) \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) & \text{if } 0 \leq p_i - p_o \leq e \text{ and } p_o < v - e, \\
2(1 - \alpha - \beta) \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) & \text{if } p_i - p_o > e \text{ and } p_o < v - e. 
\end{cases}
\]

Depending on the relationship between \( p_o \) and \( p_i \), the demand segmentation takes different forms and as is shown above, there are five possible scenarios. Compared to Case 1, it is clear that adopting 3D printing technology online has a significant impact on customers’ purchase choices and results in very different online and in-store demands.

On the other hand, the production mode remains the same as in the traditional system, that is, BTO for meeting online demand and BTS for meeting in-store demand. As a result, the firm still holds finished goods inventory for each product type in the store. Therefore, given the new online and in-store demand segmentation specified above, it is easy to verify that the firm’s total profit retains the same structure as in Case 1, i.e.,

\[
\Pi(n, p_o, p_i) = (p_o - c)d_o(n, p_o, p_i)\mu + (p_i - c)d_i(n, p_o, p_i)\mu n - ch(z^\dagger(p_i))\sigma n \sqrt{d_i(n, p_o, p_i)} - sn - k, (4)
\]

where the only difference is the fixed cost \( k \) for adopting 3D printing online. Let \( n^\dagger, p_o^\dagger, p_i^\dagger \) be the optimal solution that maximizes (4), and \( \Pi^\dagger \) be the resulting optimal profit when \( n \geq 1 \), where we use superscript “\( \dagger \)” to denote the optimal decisions and outcomes in the 3D printing online case. Similar to Case 1, the optimal safety stock factor for the in-store channel in this case is

\[
z^\dagger(p_i^\dagger) = \Phi^{-1} \left( 1 - \frac{c}{p_i^\dagger} \right).
\]

In case 2, when \( n = 0 \) (i.e., no product is offered in the in-store channel), the firm’s optimal profit from the online channel can be shown as

\[
\Pi_o^\dagger = \max((v - c)\alpha \mu, (v - e - c)(\alpha + \beta)\mu) - k.
\]
In the following proposition, we characterize structural properties of the optimal strategy under
3D printing online.

**Proposition 2.** Under 3D printing online, the following hold:
(i) If \( \Pi^1 \geq \Pi^2 \) and \( \Pi^1 \geq 0 \), the optimal strategy satisfies \( p^*_\uparrow \leq p^*_{\uparrow o} + e \) and \( p^*_i = v - \frac{e}{2n} \). The optimal
customer coverage is always full circle.
(ii) If \( \Pi^2 > \Pi^1 \) and \( \Pi^2 \geq 0 \), the product is only offered in the online channel, i.e., \( n^\uparrow = 0 \) (and \( p^*_i \) is irrelevant); \( p^*_{\uparrow o} = v \) if \( e > \frac{(v-c)d\mu-k}{(\alpha+\beta)\mu} \) and \( p^*_{\uparrow o} = v - e \) otherwise.
(iii) Otherwise, no product is offered in either channel.

Proposition 2 shows that the system with 3D printing online behaves significantly differently
from the traditional system. First, depending on the profit outcome, the firm can either sell through
both channels or completely shut off the in-store channel. Second, when both channels are used,
unlike under the traditional system, the optimal online price does not have to be lower than or
equal to the optimal in-store price. Even if the online price is lower than the in-store price, the
difference between the two prices does not exceed \( e \). In this scenario, the production mode is BTO
for the online channel and BTS for the in-store channel. Thus, same as the traditional system,
the firm has the incentive to offer a lower online price because of the elimination of inventory
mismatching cost in the online channel. However, using 3D printing online significantly increases
the product offering in the online channel, and hence allows the firm to increase the price offered
in the online channel. Thus, the online price can exceed the in-store price in this case. Third, when
both channels are used, unlike the traditional system, the optimal product offering is associated
with the in-store price, but not the online price. Moreover, in this case, it is interesting to note
that no customers are left unserved.

As in Case 1, to gain further insights, we analyze the special scenario with \( \sigma = 0 \) where we
can obtain closed-form solutions. A detailed analysis of the scenario can be found in Appendix
B. The following proposition summarizes the comparison results between this case and Case 1 for
sufficiently small \( \sigma \) when both channels are used:

**Proposition 3.** The following results hold when \( \sigma \) is sufficiently small:
(i) \( n^\uparrow < n^* \); \( p^\uparrow _o > p^*_o \); \( p^\uparrow _i < p^*_i \); \( z^\uparrow (p^\uparrow _i) < z^* (p^*_i) \).
(ii) There exists a threshold \( \tilde{e}_1 \geq 0 \), such that online demand proportion \( d_o(n^\uparrow ,p^\uparrow _o,p^\uparrow _i) < d_o(n^*,p^*_o,p^*_i) \) and in-store demand proportion \( n^\uparrow d_i(n^\uparrow ,p^\uparrow _o,p^\uparrow _i) > n^*d_i(n^*,p^*_o,p^*_i) \) if and only if
\( e > \tilde{e}_1 \).
(iii) There exists a threshold \( \bar{k} > 0 \), such that \( \Pi^1 \geq \Pi^* \) if and only if \( k \leq \bar{k} \). \( \Pi^1 > \Pi^* \) when \( k = 0 \).
Moreover, \( \Pi^1 - \Pi^* \) is increasing in both \( t \) and \( s \).
Proposition 3 reveals important insights regarding the impact of adopting 3D printing online. As Part (i) indicates, adopting 3D printing online enables the firm to charge a price premium online due to perfect customization. At the same time, the in-store product offering reduces, and correspondingly, the in-store price decreases. Following the price change, the optimal safety stock factor decreases for meeting in-store demand (due to the decreased unit underage cost).

Part (ii) of Proposition 3 characterizes how the firm’s demand segmentation changes. With 3D printing online, which allows for perfect customization through the online channel, customers’ channel choices become more complicated. Type I customers need to trade off the improved fit from purchasing online and the reduced price from purchasing in-store. If a Type I customer cannot find a product that is close enough to her ideal type from the firm’s in-store offerings, then she purchases online to pursue the improved fit; otherwise she purchases in-store to take advantage of the reduced price. Type II customers face the same trade-off as Type I customers, but they also need to factor in their waiting cost $e$ from purchasing online. Thus, compared to Type I customers, a smaller proportion of Type II customers choose to purchase online. If the waiting cost $e$ is high, not many Type II customers would want to switch to purchase online, so the firm’s total online demand becomes lower while the total in-store demand becomes higher with 3D printing online.

On the other hand, if the waiting cost $e$ is low, then Type II customers who switch to purchase online outnumber the Type I customers who switch to purchase in-store, so the firm’s total online demand becomes higher while the total in-store demand becomes lower with 3D printing online.

Part (iii) of Proposition 3 states that the firm achieves a higher profit with 3D printing online as long as the fixed cost of technology adoption is not too high. Additionally, if we ignore the fixed cost as a sunk cost, the operating profit is always improved when the firm adopts 3D printing online. Adopting 3D printing online creates two benefits for the firm. First, the 3D printing technology allows the firm to achieve perfect customization and eliminate the fit cost for the customers, and hence enables the firm to charge a price premium for products sold online. 4 Second, adopting 3D printing online reduces the number of product types that the firm offers in the store, and hence reduces the firm’s production setup cost. Therefore, the firm achieves another benefit of setup cost reduction with 3D printing. These two benefits are exemplified by the result that the profit improvement with 3D printing online increases in both the customer fit cost parameter $t$ (which corresponds to the benefit of perfect customization) and the setup cost parameter $s$ (which corresponds to the benefit of setup cost reduction).

Overall, the impact of adopting 3D printing online stems mainly from technology substitution, i.e., the traditional production technology is replaced by a better 3D printing technology. Such

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4 This is consistent with industry anecdotes we have learned through our interactions with practitioners, e.g., a 3D printed customized coffee mug can be priced at an exuberant rate of $2,000 in an online store of a major U.S. retailer.
technology substitution leads to the variety effect, enabled by 3D printing’s natural elimination of the production setup cost, and allows the firm to offer perfect customization and charge a price premium. As we have seen, the induced online and in-store demands are changed significantly due to the technology substitution. However, the supply chain structure remains the same, i.e., BTO for online demand and BTS for in-store demand. Therefore, the profit-maximization problems share the same structure and trade-off as shown in (3) and (4). We next examine the impact of adopting 3D printing in-store.

4.3. Case 3: 3D Printing In-Store

In Case 3 (3D printing in-store; see Figure 3), the firm uses the 3D printing technology to offer perfectly customized products in both channels. Same as before, we first characterize the customer choices as follows.

- Type I customers: Their utility from purchasing online is \( v - p_o \), and their utility from purchasing in-store is \( v - p_i \). Thus, Type I customers purchase online if \( p_i - p_o \geq 0 \), and purchase in-store if \( p_i - p_o < 0 \).
- Type II customers: Their utility from purchasing online is \( v - p_o - e \), and their utility from purchasing in-store is \( v - p_i \). Thus, Type II customers purchase online if \( p_i - p_o \geq e \), and purchase in-store if \( p_i - p_o < e \).
- Type III customers: Their utility from purchasing in-store is \( v - p_i \), and all Type III customers purchase in-store.

Based on the customer choices, we obtain that the proportion of customers purchasing online is

\[
d_o(p_o, p_i) = \begin{cases} 
\alpha + \beta & \text{if } p_i - p_o \geq e, \\
\alpha & \text{if } 0 \leq p_i - p_o < e, \\
0 & \text{if } p_i - p_o < 0,
\end{cases}
\]

and the (total) proportion of customers purchasing in-store is

\[
d_i(p_o, p_i) = \begin{cases} 
1 - \alpha - \beta & \text{if } p_i - p_o \geq e, \\
1 - \alpha & \text{if } 0 \leq p_i - p_o < e, \\
1 & \text{if } p_i - p_o < 0.
\end{cases}
\]

Depending on the relationship between \( p_o \) and \( p_i \), the demand segmentation takes different forms and as is shown above, there are three possible scenarios. Note that with 3D printing in both channels, which enables perfect customization in both channels, the firm’s optimization problem does not involve the decision variable \( n \). Compared to Cases 1 and 2, it is clear that adopting 3D printing in both channels has a significant impact on customers’ purchase choices.

With 3D printing in both channels, the firm’s production is BTO for meeting both online and in-store demands. However, as it is unlikely that suppliers will be co-located with brick-and-mortar
stores, we assume that the firm still needs to hold raw material inventory in the store (to meet the total in-store demand) and decide the inventory order quantity for raw material before demand realization. The total online demand $D_o$ follows a normal density $f_o(\cdot)$ with mean $\mu_o(p_o, p_i) = d_o(p_o, p_i)\mu$ and standard deviation $\sigma_o(p_o, p_i) = \sigma \sqrt{d_o(p_o, p_i)}$. The total in-store demand $D_i$ follows a normal density $f_i(\cdot)$ with mean $\mu_i(p_o, p_i) = d_i(p_o, p_i)\mu$ and standard deviation $\sigma_i(p_o, p_i) = \sigma \sqrt{d_i(p_o, p_i)}$. The firms’s total profit is

$$\Pi(p_o, p_i, q) = \Pi_o(p_o, p_i) + \Pi_i(p_o, p_i, q) - k - k',$$

where $\Pi_o(p_o, p_i)$ and $\Pi_i(p_o, p_i, q)$ are the profits from the online and in-store channels, respectively, and $k$ and $k'$ are the respective fixed costs for adopting 3D printing in those two channels. Since both channels adopt 3D printing, there are no predetermined product types and thus no setup cost $s$ is incurred compared to Cases 1 and 2.

The firm’s profit from the in-store channel can be written as

$$\Pi_i(p_o, p_i, q) = (p_i - c_r)E[\min(D_i, q)] - c_rq$$

$$= (p_i - c)\int_{-\infty}^{\tau}(q-x)f_i(x)\,dx - (p_i - c)\int_{q}^{\infty}(x-q)f_i(x)\,dx.$$

In this case, the raw material needs to be ordered in advance and shipped to the store. Thus, the raw material cost $c_r$ is incurred before demand realization, and the production cost $c_p$ is incurred after demand realization. Therefore, the unit overage cost is $c_r$, and the unit underage cost is $p_i - c$. It follows from standard newsvendor analysis that the optimal order quantity is $q^\dagger(p_o, p_i) = \mu_i(p_o, p_i) + z^\dagger(p_i)s_i(p_o, p_i)$ where

$$z^\dagger(p_i) = \Phi^{-1}\left(1 - \frac{c_r}{p_i - c_p}\right).$$

Substituting in $q = q^\dagger(p_o, p_i)$ and leveraging the normal distribution property, one can show that the online profit function reduces to

$$\Pi_o(p_o, p_i) = (p_o - c)d_o(p_o, p_i)\mu - c_rh(z^\dagger(p_i))\sigma \sqrt{d_o(p_o, p_i)},$$

where the expected inventory mismatch (i.e., overage and underage) cost is proportional to the raw material cost $c_r$ (because the raw material shipped to store is used as safety stock). Note that this cost term is different from that in (2) of Case 1 (as well as Case 2) because the in-store channel switches from BTS to BTO.

Since the online channel is also BTO, following analogous analysis in Cases 1 and 2, we can express the firm’s profit from the online channel as

$$\Pi_o(p_o, p_i) = (p_o - c)d_o(p_o, p_i)\mu.$$
From (5) and (6), the firm’s total profit under the 3D printing in-store case is

\[ \Pi(p_o, p_i) = (p_o - c_d) d_o(p_o, p_i) \mu + (p_i - c_d) d_i(p_o, p_i) \mu - c_e \sqrt{d_i(p_o, p_i)} - k - k'. \]  

(7)

Let \( p^*_o \) and \( p^*_i \) be the optimal solution that maximizes (7), and \( \Pi^* \) be the resulting optimal profit, where we use superscript “*” to denote the optimal decisions and outcomes in the 3D printing in-store case. In the following proposition, we characterize structural properties of the optimal strategy under 3D printing in-store.

**Proposition 4.** Under 3D printing in-store, the following hold:

(i) If \( \Pi^* \geq 0 \), the optimal strategy is \( p^*_o \leq p^*_i = v \); \( p^*_o = v \) if \( e > c_e \sqrt{\frac{h(z^*(v))}{(\alpha + \beta)\mu}} \), and \( p^*_o = v - e \) otherwise. The optimal customer coverage is always full circle.

(ii) If \( \Pi^* < 0 \), no product is offered in either channel.

Proposition 4 states that when the firm uses 3D printing in both channels, the optimal online price is lower than or equal to the optimal in-store price. Recall that this is also true under the traditional system (Proposition 1). When the firm uses 3D printing in both channels, the firm offers the same degree of customization in both channels, and the production mode is BTO in both channels. However, BTO in the in-store channel still involves raw material inventory mismatch cost in the store. Thus, when the demand coefficient of variation \( \sigma/\mu \) is relatively large or when the online waiting cost \( e \) is relatively small, it is optimal for the firm to offer a lower price in the online channel to attract more demand from the in-store channel to the online channel. Proposition 4 also shows that due to perfect customization, the optimal in-store price is always equal to the customers’ valuation for the ideal product type \( v \), while the optimal online price can be either \( v \) or \( v - e \). Moreover, same as in Case 2, when both channels are used, no customers are left unserved. The following proposition summarizes the comparison results between this case and Cases 1 and 2 for sufficiently small \( \sigma \) when both channels are used:

**Proposition 5.** The following results hold when \( \sigma \) is sufficiently small:

(i) \( p^*_o > p^*_d > p^*_s \); \( p^*_i > p^*_d > p^*_s \); \( z^*(p^*_o) > z^*(p^*_i) \).

(ii) There exists a threshold \( \bar{e}_d \geq 0 \), such that online demand proportion \( d_o(p_o, p_i) > d_o(n^1, p_o, p_i) \)

and in-store demand proportion \( d_i(p_o, p_i) < n^1 d_i(n^1, p_o, p_i) \) if and only if \( e > \bar{e}_d \).

(iii) There exists a threshold \( \bar{k}' > 0 \), such that \( \Pi^d \geq \Pi^s \) if and only if \( k' \leq \bar{k}' \). \( \Pi^d \geq \Pi^* \) when \( k' = 0 \).

Moreover, \( \Pi^d - \Pi^s \) and \( \Pi^d - \Pi^* \) are increasing in both \( t \) and \( s \).

Proposition 5 reveals important insights regarding the impact of adopting 3D printing in both channels. Part (i) states that using 3D printing in both channels allows the firm to charge the highest price among all three cases. Moreover, the optimal safety stock factor is also the highest...
among the three cases. The increase in the safety stock factor for the in-store channel is caused by both the increased unit underage cost (due to increased in-store price) and the reduced unit overage cost (due to production mode being changed from BTS to BTO for the in-store channel).

Part (ii) shows that when the firm also adopts 3D printing in-store, compared to the case of only using 3D printing online, the demand segmentation between the two channels shifts in an opposite way of Proposition 3(b). In this case, the firm offers perfect customization in both channels, which leads to more even pricing between the two channels. Thus, unlike the 3D printing online case where the degree of product customization is asymmetric between the two channels, the demand segmentation reverses to one similar to that under the traditional system (where the degree of product customization is symmetric between the two channels).

Part (iii) states that a firm that is already using 3D printing online will achieve a higher profit with 3D printing in-store as long as the fixed cost of technology adoption is not too high. Additionally, if we ignore the fixed cost as a sunk cost, the operating profit is always improved when the firm adopts 3D printing in-store. Using 3D printing in-store strengthens the benefits of using 3D printing online. First, the firm achieves perfect customization in both channels, and hence charges higher price premiums in both channels compared to the previous two cases. Second, because the 3D printing technology naturally allows for flexible product types, the firm completely eliminates the setup cost and increases product variety in both channels. The benefit of perfect customization becomes stronger when customers’ fit cost parameter $t$ is higher, and the benefit of setup cost reduction becomes stronger when production setup cost parameter $s$ is higher under the traditional production technology.

Moreover, the impact of 3D printing in-store stems not only from technology substitution (as discussed in Case 2), but also from supply chain structural change. With 3D printing in-store, the production process for the in-store channel changes from BTS to BTO, which naturally leads to the postponement effect. This effect allows the firm to achieve both inventory pooling and overage cost reduction. To see this, recall from (2) and (5) that the expected overage and underage cost terms for the in-store channel in Cases 1 and 3 are

$$ch(z^*(p^*_i))\sigma n^* \sqrt{d_i(n^*, p^*_o, p^*_i)}, \quad \text{and} \quad c_r h(z^i(p^i))\sigma \sqrt{d_i(p^i_o, p^i)}.$$

We can write the difference between these two terms as follows:

$$\underbrace{ch(z^*(p^*_i))\sigma \sqrt{d_i(n^*, p^*_o, p^*_i)}}_{\text{cost reduction from inventory pooling}} + \underbrace{c_r h(z^i(p^i))\sigma \sqrt{d_i(p^i_o, p^i)}}_{\text{overage cost reduction}} \left[ h(z^*(p^*_i)) \sqrt{n^*} - h(z^i(p^i)) \sqrt{n^*} \right],$$

where $d_i = d_i(n^*, p^*_o, p^*_i)$ and $d_i = d_i(p^i_o, p^i)$. The first term of the above expression captures the cost reduction from inventory pooling, which is proportional to the square root of the number of
product types that the firm offers in Case 1 (Eppen 1979, Corbett and Rajaram 2006). This is due to the fact that, with BTO for the in-store channel in Case 3, the firm only needs to make raw material inventory decision to meet total in-store demand, whereas in Case 1 the firm needs to make finished goods inventory decision for each product type under BTS for the in-store channel.\(^5\) The second term captures the overage cost reduction. With 3D printing in-store, the firm only makes the in-store production when demand occurs. Thus, the firm saves the production cost \(c_p\) should demand turn out to be lower than supply. This reduces the overage cost for the firm and helps improve profitability. The remaining terms correspond to the inventory cost difference that is driven by price differences between the two cases. These insights also hold when comparing Cases 2 and 3, as Case 2 also uses BTS for the in-store channel.

Another important implication of supply chain structural change from 3D printing in-store is that it will require a new supplier-retailer relationship. So far, we have assumed that the firm owns the brick-and-mortar store. In practice, this may not always be the case. For example, Nike relies heavily on its brick-and-mortar retail partners (e.g., Footlocker) for selling its products to customers. Thus, the case of 3D printing in-store would represent a drastic departure from the current supplier-retailer relationship, as the retailer now does on-site production in the form of 3D printing. A question of interest is how to coordinate the supply chain by incentive and contract design under such a new relationship. We will briefly explore this topic below.

### 4.4. Contracting for 3D Printing In-Store

In this section, we take a step further to consider a (decentralized) 3D printing in-store case in which the store is owned by an independent retailer and the firm (who now becomes the supplier) provides raw material to the retailer. Our goal is to study how to coordinate such a supply chain, i.e., given the product pricing decisions, inducing the same inventory order quantity as the integrated system studied previously, so that the supply chain profit is maximized. We shall focus on a scenario where the supplier owns the raw material and makes inventory decisions, which is highly plausible as the supplier has expertise in raw material procurement and supplier computer-aided design (CAD) changes may require raw material changes. In this scenario, the retailer could be considered as the supplier’s co-production partner who is responsible for sales as well as production.

\(^5\) Note that in our model, we assume a single raw material type for analytical simplicity. We would like to point out that incorporating multiple raw material types does not reduce the pooling benefit of 3D printing. In that case, although the firm needs to choose an inventory quantity for each raw material type, because the demands corresponding to different raw material types are perfectly positively correlated (which are all equal to the total demand for all product types, assuming that in order to produce one product, the firm needs one unit of inventory from each raw material type), the variety of raw materials does not reduce the pooling effect at all. Thus, the complexity of the product design (as measured by the number of raw material types needed) does not magnify the firm’s inventory cost under 3D printing.
Recall from §4.3 that the supply chain profit from the in-store channel (given the optimal product prices) can be written as

\[ \Pi_i(q) = (v - c_p)E[\min(D_i, q)] - c_r q, \]
and the channel-profit-maximizing safety stock factor is

\[ z^+ = \Phi^{-1} \left( 1 - \frac{c_r}{v - c_p} \right). \] (8)

Recall from Proposition 4 that the optimal in-store price is \( p^+_i = v \). For ease of notation, we shall drop the subscript “\( i \)” from all corresponding functions and variables in this section.

In the decentralized supply chain, the supplier makes inventory decisions, while the two parties negotiate on the wholesale price \( w \) for the finished goods. The raw material cost is incurred by the supplier, while the production cost is incurred by the retailer. The supplier’s profit is

\[ \Pi_S(q, w) = wE[\min(D, q)] - c_r q, \]
and the retailer’s profit is

\[ \Pi_R(q, w) = (v - c_p - w)E[\min(D, q)]. \]

Under the wholesale contract, for any wholesale price \( w \), the supplier’s optimal safety stock factor is

\[ z^w_S = \Phi^{-1} \left( 1 - \frac{c_r}{w} \right). \] (9)

The subscript “\( S \)” indicates supplier managing the inventory. Thus, by comparing (9) to (8), we can see that as long as \( w < v - c_p \), there exists loss of efficiency in the decentralized supply chain.

When the supplier controls the raw material inventory, the wholesale contract is similar to the pull contract studied by Cachon (2004), with the difference being that the retailer’s production cost is incurred after demand realization. Under a pull system, traditional contracts such as buyback and revenue-sharing are difficult to implement in principle, because transactions between the retailer and the supplier occur after demand realization. Therefore, we introduce a new type of contract that has not been studied by previous contracting literature – cost-sharing contract. In this case, the retailer shares a fraction of the supplier’s raw material cost. The retailer still earns the profit and pays the wholesale price after demand realization. Let \( \beta \) denote the retailer’s share of raw material cost. The supplier and the retailer negotiate on \( (w, \beta) \). Under the cost-sharing contract, the supplier’s profit is

\[ \Pi_S^c(q, w, \beta) = wE[\min(D, q)] - (1 - \beta)c_r q, \]
where the superscript “\( c \)” indicates cost-sharing contract, and the retailer’s profit is

\[ \Pi_R^c(q, w, \beta) = (v - c_p - w)E[\min(D, q)] - \beta c_r q. \]
Proposition 6. Suppose that the supplier controls the raw material inventory. The supply chain can be coordinated under the set of cost-sharing contracts \((w, \beta)\) with

\[
\beta = 1 - \frac{w}{v - c_p}.
\]

(10)

Under these contracts, the supplier’s optimal inventory order quantity is equal to the optimal quantity in the integrated supply chain, and the supplier’s optimal profit is \((1 - \beta) \cdot \Pi(q^\dagger)\).

Proposition 6 shows that cost-sharing contracts can coordinate the supply chain of 3D printing in-store where the supplier controls the raw material inventory. Moreover, the supplier’s share of the supply chain profit is equal to its share of the raw material cost. This cost-sharing contract complements the well-known supply-chain-coordinating buy-back and revenue-sharing contracts in the literature. A coordinating cost-sharing contract is also easy to implement. For example, the retailer can be responsible for handling the distribution of raw materials and hence pay for all or part of the distribution cost. We project that with the emerging trend of on-demand production such as 3D printing in-store, cost-sharing contracts between suppliers and retailers will start to gain traction in practice. For completeness, we also include the analysis for the (less likely) scenario where the retailer owns the raw material and makes inventory decisions in Appendix C.

5. Numerical Analysis

While we are able to characterize the structural properties of the optimal strategy under each case (Propositions 1, 2, and 4), the analytical comparison results (Propositions 3 and 5) are obtained when \(\sigma\) is sufficiently small (e.g., functional products such as coffee mugs). For cases with relatively large \(\sigma\) (e.g., innovative products such as hi-fashion sneakers), the comparison is, however, analytically intractable. To gain additional insights, we numerically examine the impact of 3D printing with a general \(\sigma\) in this section. Additionally, in our numerical study, we allow \(n\) to be a non-negative integer.

In Figure 4, we compare the firm’s optimal strategies under Case 1 (traditional system), Case 2 (3D printing online), and Case 3 (3D printing in-store). The results are presented as functions of \(\sigma\). Figure 4(a) shows that when the firm adopts 3D printing online, its optimal in-store product offering becomes smaller (i.e., \(n^\dagger \leq n^*\)), confirming the insight from Proposition 3(i). It is worth commenting that because of the integer constraint for \(n\), the inequality may not be always strict. However, as Figure 4(a) indicates, the insight from Proposition 3(i) regarding the impact of adopting 3D printing online on the firm’s in-store product offering carries through to large \(\sigma\), and is hence valid for both functional and innovative products. Additionally, we observe that the firm may be better off shutting off the in-store channel in Case 2 if demand is extremely uncertain. Specifically, with the parameters in Figure 4, \(n^\dagger\) would be zero if \(\sigma \geq 0.56\).
Figure 4 Comparison of optimal strategies in Case 1 (traditional system), Case 2 (3D printing online), and Case 3 (3D printing in-store) (Parameters: $\mu = 1$, $\alpha = 0.2$, $\beta = 0.5$, $v = 1$, $e = 0.05$, $t = 0.5$, $c_r = 0.1$, $c_p = 0.1$, $s = 0.005$, $k = 0$, $k' = 0$)

Figure 4(b) shows the optimal prices in each case. We know from Proposition 1 that $p_o^* \leq p_i^*$ in Case 1. Figure 4(b) further shows that when $\sigma$ is small ($\sigma \leq 0.18$), $p_o^* = p_i^*$; otherwise, $p_o^* < p_i^*$. As $\sigma$ becomes large enough, the inventory cost difference between the two channels (due to BTO in the online channel and BTS in the in-store channel) becomes significant enough, so the prices start to differ. Additionally, in the example provided in Figure 4(b), as $\sigma$ increases above 0.18 and $p_o^*$ and $p_i^*$ start to diverge, $n^*$ initially increases and then decreases. This shows that $n^*$ is not necessarily monotone decreasing in $\sigma$.

We know from Proposition 2 that $p_i^1 \leq p_o^1 + e$ in Case 2. Figure 4(b) further shows that when $\sigma$ is small ($\sigma < 0.09$), $p_i^1 < p_o^1$; otherwise, $p_o^1 \leq p_i^1 \leq p_o^1 + e$. When the firm uses 3D printing online, while the firm may have the incentive to charge a higher price in the online channel due to perfect customization, the firm may also have the incentive to charge a lower price in the online channel to attract more demand from the in-store channel due to the elimination of inventory mismatch cost. This trade-off is at a clear display with curves of $p_o^1$ and $p_i^1$. When $\sigma$ is small, the inventory mismatch cost is small, so the perfect customization effect dominates, and the online price is higher than the
in-store price. On the other hand, when \( \sigma \) is large, the inventory mismatch cost dominates, and the online price is lower than the in-store price. Therefore, after the firm adopts 3D printing online, the relationship between the prices in the two channels may be different for functional products (i.e., \( \sigma \) is small) and innovative products (i.e., \( \sigma \) is large).

We know from Proposition 4 that \( p^\dagger_o \leq p^\dagger_i \) in Case 3, and \( p^\dagger_i \) is always equal to \( v \) while \( p^\dagger_o \) can be either \( v \) or \( v - e \). In the example provided in Figure 4(b), \( p^\dagger_o = p^\dagger_i = v \), which is commonly observed in other examples as well, indicating the power of 3D printing in allowing the firm to charge the highest possible price in both channels.

Moreover, we can compare the optimal prices under the three cases in Figure 4(b). First, Figure 4(b) shows that after the firm adopts 3D printing online, the in-store price will decrease (i.e., \( p^\dagger_i \leq p^\ast_i \)) in response to the reduced in-store product offering, while for small \( \sigma \) (\( \sigma < 0.09 \)), the online price will increase (i.e., \( p^\dagger_o > p^\ast_o \)). Second, Figure 4(b) shows that after the firm also adopts 3D printing in-store, both the online price and the in-store price will increase compared to the other two cases. These observations again confirm the insights from Propositions 3(i) and 5(i).

In Figure 4(c), we compare the optimal in-store safety stock factors under the three cases. First, Figure 4(c) shows that after the firm adopts 3D printing online, the safety stock factor will become lower for the in-store channel (i.e., \( z^\dagger_i \leq z^\ast_i \)). However, the numerical result indicates that the change in safety stock factor is not drastic, because in both Case 1 and Case 2, the safety stock factor corresponds to inventory of finished goods. Second, Figure 4(c) shows that after the firm also adopts 3D printing in-store, the safety stock factor will increase drastically compared to the previous two cases. Adopting 3D printing in-store fundamentally changes the supply chain structure and the in-store production is BTO instead of BTS. In this case, the safety stock factor corresponds to inventory of raw materials.

In Figure 5, we compare the demand segmentations under the optimal strategies in the three cases. The results are presented as functions of customers’ online purchase waiting cost \( e \). In Figures 5(a) and 5(b), \( \sigma \) is small (\( \sigma = 0.05 \)). In Cases 1 and 3, under the optimal strategy, all Type I customers purchase online and all Type II and Type III purchase in-store; the firm’s demand segmentation is independent of \( e \). In Case 2, the online (in-store) demand is decreasing (increasing) in \( e \). Thus, the results of demand segmentation comparison are consistent with Propositions 3(ii) and 5(ii). Moreover, in Figures 5(c) and 5(d), \( \sigma \) is relatively large (\( \sigma = 0.15 \)). As Figure 5(c) indicates, in Case 1, Type II customers purchase online instead of in-store when \( e \leq 0.43 \). That is, when the in-store channel’s disadvantage of inventory mismatch cost is significant and Type II customers’ online purchase waiting cost is not too high, the firm should steer Type II customers to the online channel. This change would reverse the result of demand segmentation comparison between Cases 1 and 2 for small \( e \) when \( \sigma \) is large. Overall, in this case (large \( \sigma \)), when 3D printing
Figure 5 Comparison of demand segmentations under optimal strategies in Case 1 (traditional system), Case 2 (3D printing online), and Case 3 (3D printing in-store) (Parameters: $\mu = 1$, $\alpha = 0.2$, $\beta = 0.5$, $v = 1$, $t = 0.5$, $c_r = 0.1$, $c_p = 0.1$, $s = 0.005$, $k = 0$, $k' = 0$)

is adopted online, customers are steered from the in-store channel to the online channel if and only if $e$ is medium. However, the demand segmentation comparison result between Cases 2 and 3 carries through to the large $\sigma$ case.

Finally, in Figure 6, we compare the firm’s optimal profits under the three cases. We present the profit difference between Case 2 and Case 1 (i.e., $\Pi^\dagger - \Pi^*$) and the profit different between Case 3 and Case 2 (i.e., $\Pi^\ddagger - \Pi^\dagger$) separately. Moreover, we present the profit differences both as functions of $t$ and as functions of $s$, for different levels of $\sigma$. In all examples, $\Pi^\dagger - \Pi^*$ and $\Pi^\ddagger - \Pi^\dagger$ show increasing trends in both $t$ and $s$. Note that because we impose integer constraint for $n$ in our numerical study, the profit functions in Cases 1 and 2 become piecewise functions and the profit difference may be slightly decreasing in certain regions. However, if we relax the integer constraint for $n$, the profit difference will be always increasing in $t$ and $s$, verifying the benefits of perfect customization (measured by $t$) and setup cost reduction (measured by $s$) from adopting 3D printing online and in-store. Moreover, note that in the examples shown in Figure 6, $k = 0$ and $k' = 0$. If $k > 0$, the $\Pi^\dagger - \Pi^*$ curves will move down by the amount of $k$, thus one can easily see
the threshold of $k$ below which adopting 3D printing online is profitable. Similarly, if $k' > 0$, the $\Pi^\dagger - \Pi^\ddagger$ curves will move down by the amount of $k'$, thus one can easily see the threshold of $k'$ below which adopting 3D printing in-store is profitable.

As discussed above, when adopting 3D printing online, there is a trade-off between charging a higher price online (due to perfect customization) and charging a lower price online (to reduce inventory mismatch cost in the in-store channel). As a result, from Figures 6(a) and 6(c), we observe that the profit improvement from Case 2 is not necessarily always increasing in $\sigma$. By contrast, recall from §4.3 that, when the firm adopts 3D printing in-store, the postponement of production in the store enables the firm to achieve inventory pooling and overage cost reduction. The value of postponement comes from the ability to hedge against demand uncertainty. Thus, when the demand is more uncertain, the benefit of postponement becomes stronger. From Figures 6(b) and 6(d), we observe that the profit improvement from Case 3 is always increasing in $\sigma$. It is also worth commenting that as Figure 6(c) indicates, for functional products, the profit improvement from the substitution effect is not very sensitive to $s$ compared to innovative products. Moreover, the scale
of \( \Pi^1 - \Pi^1 \) is greater than that of \( \Pi^1 - \Pi^* \), showing that the profit improvement from the structural effect is much greater than that from the substitution effect. Overall, our numerical findings are consistent with all results in Propositions 3 and 5 for functional products (i.e., \( \sigma \) is small), and most results carry through to innovative products (i.e., \( \sigma \) is large) as well.

6. Concluding Remarks

In this paper, we have developed an integrated model to analyze the impact of 3D printing on retail product offering, prices for online and in-store channels, as well as inventory decisions. Table 1 summarizes the main insights we have obtained from our model analysis.

<table>
<thead>
<tr>
<th>Functional Products (Small ( \sigma ))</th>
<th>Innovative Products (Large ( \sigma ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Online price same as in-store price</td>
<td>• Online price less than in-store price</td>
</tr>
<tr>
<td>• Online price determines product variety</td>
<td>• Online price determines product variety</td>
</tr>
<tr>
<td>• No customers are left unserved</td>
<td>• Some customers are left unserved</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Traditional System with Dual Channels (Case 1)</th>
<th>Substitution effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Online price greater than in-store price</td>
<td>• In-store product offering decreases</td>
</tr>
<tr>
<td>• In-store price determines product variety</td>
<td>• Online price increases, in-store price decreases</td>
</tr>
<tr>
<td>• No customers are left unserved</td>
<td>• In-store safety-stock factor (for FG) decreases</td>
</tr>
<tr>
<td></td>
<td>• Customers are steered from in-store to online iff online purchasing waiting cost is small (if ( \sigma ) is small)/medium (if ( \sigma ) is large)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3D Printing Online (Case 2)</th>
<th>Structural effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Online price same as in-store price</td>
<td>• Both online and in-store prices are highest</td>
</tr>
<tr>
<td>• No customers are left unserved</td>
<td>• In-store safety-stock factor (for RM) is highest</td>
</tr>
<tr>
<td></td>
<td>• Customers are steered from in-store to online iff online purchasing waiting cost is large</td>
</tr>
<tr>
<td></td>
<td>• Cost-sharing contracts can help coordinate the in-store channel in a decentralized setting</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3D Printing In-Store (Case 3)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Online price same as in-store price</td>
<td></td>
</tr>
<tr>
<td>• No customers are left unserved</td>
<td></td>
</tr>
<tr>
<td>• Online price less than in-store price</td>
<td></td>
</tr>
<tr>
<td>• No customers are left unserved</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 Summary of effects of 3D printing.

Overall, we have seen two major effects of 3D printing: the substitution effect and the supply chain structural effect. Adopting 3D printing for the online channel gives rise to a substitution effect of technological innovation. Such technology substitution leads to the variety effect, enabled by 3D printing’s natural elimination of the production setup cost, and allows the firm to offer perfect customization and charge a price premium for online customers. At the same time, the firm offers a smaller product variety in the store at a reduced price. Moreover, when 3D printing is used in the online channel, the effect of enhanced customization in the online channel has to be
balanced against the opposite effect induced by the BTO/BTS difference of the online and in-store channels. Specifically, for functional products with low demand uncertainty, the firm should charge a price premium for the 3D-printed products. However, for innovative products with high demand uncertainty, the firm should set the online price lower than the in-store price to attract more demand to the online channel. Thus, different pricing strategies are needed for different product characteristics in this case.

When 3D printing is used at stores, the substitution effect is also compounded by the structural effect due to the change in the supply chain structure. With 3D printing in-store, the store demands are now served by BTO instead of BTS, which naturally leads to the postponement effect. This effect allows the firm to achieve both inventory pooling and overage cost reduction. Moreover, with perfect customization in both channels, the firm can charge a full price premium for at least the in-store channel. Another important implication of supply chain structural change from 3D printing in-store is that it will require a new supplier-retailer relationship. We show that cost-sharing contracts can coordinate the supply chain of 3D printing in-store where the supplier controls the raw material inventory.

Our numerical results also suggest that the magnitude of profit improvement of structural effect is much greater than that of substitution effect, which is consistent with the observation made by Lee (2007) that structural changes are often deeper than substitution from technological innovation. Therefore, in the case of 3D printing technology, its impact to the supply chain will be more fully felt when it is adopted by retailers in addition to manufacturers.

Our work is a first step toward understanding the impacts of 3D printing technology on retail supply chains. We expect that several further extensions of our current model may bear fruitful and interesting results. First, for analytical tractability, we have assumed a stylized circular city model for consumer preferences over horizontally-differentiated products. It would be interesting to explore how the optimal decisions would change under other consumer choice models such as the multinomial logit model. Second, in our model, we do not consider the possibility of product returns. With traditional technology, a returned product can still be resold at a regular price or at a discount. With 3D printing, products are all custom-made, so it may not be easy to resell a returned product. It would be interesting to investigate how product return issues could impact firms’ profitability. Finally, we have considered a monopolistic setting. It would be interesting to explore how 3D printing can impact competition between retail supply chains.
Appendix A: Proofs of Results in Main Text

Proof of Proposition 1 We focus on \( \Pi^* \geq 0 \), because the case of \( \Pi^* < 0 \) is trivial. First, we prove \( p^*_o \leq p^*_t \) by contradiction. Suppose the opposite is true, that is, \( p^*_o > p^*_t \). Then, the optimal demand segmentation is given by \( d_o(n^*, p^*_o, p^*_t) = 0 \) and \( d_o(n^*, p^*_o, p^*_t) = 2 \min\left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) \), and the optimal profit is

\[
\Pi^* = (p^*_t - c)\mu \min \left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) 2n^* - ch(z^*(p^*_t))\gamma n^* \sqrt{2} \min \left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) - sn^*. 
\]

Now, consider another strategy \((\tilde{n}, \tilde{p}_o, \tilde{p}_t) = (n^*, p^*_o, p^*_t)\). Under this strategy, the demand segmentation is given by \( d_o(\tilde{n}, \tilde{p}_o, \tilde{p}_t) = \alpha \min\left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) 2n^* \) and \( d_o(\tilde{n}, \tilde{p}_o, \tilde{p}_t) = 2(1-\alpha) \min\left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) \), and the profit is

\[
\Pi(\tilde{n}, \tilde{p}_o, \tilde{p}_t) = (p^*_t - c)\mu \min \left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) 2n^* - ch(z^*(p^*_t))\gamma n^* \sqrt{2} (1-\alpha) \min \left( \frac{v-p^*_t}{t}, \frac{1}{2n^*} \right) - sn^*. 
\]

It is easy to see that \( \Pi(\tilde{n}, \tilde{p}_o, \tilde{p}_t) > \Pi^* \), hence contradiction. Therefore, we must have \( p^*_o \leq p^*_t \) under the optimal strategy.

We next characterize the structural property of \( n^* \). First, consider \( 0 \leq p^*_o - p^*_t \leq e \). In this case, we must have \( \frac{1}{2n^*} = \frac{v-p^*_t}{t} \) due to the following reasons. Suppose \( \frac{1}{2n^*} < \frac{v-p^*_t}{t} \). Then, the optimal profit is

\[
\Pi^* = (p^*_o - c)\alpha \mu \left( \frac{v-p^*_t}{t} \right) + (p^*_t - c)2(1-\alpha) \mu \left( \frac{v-p^*_t}{t} \right) - ch(z^*(p^*_t))\gamma \left( 2(1-\alpha) \frac{v-p^*_t}{t} - s \right) n^*. 
\]

\( \Pi^* \) is a linear function of \( n^* \) in this case. Since \( \Pi^* \geq 0 \), the terms within the bracket in the above expression must be non-negative. Then, we can increase \( n^* \) by increasing \( n \), hence contradiction. Therefore, if \( 0 \leq p^*_o - p^*_t \leq e \), we must have \( \frac{1}{2n^*} = \frac{v-p^*_t}{t} \).

Second, consider \( p^*_o - p^*_t > e \). We must have either \( \frac{1}{2n^*} = \frac{v-p^*_t}{t} \) or \( \frac{1}{2n^*} = \frac{v-p^*_t-e}{t} \) due to the following reasons. Suppose \( \frac{1}{2n^*} < \frac{v-p^*_t-e}{t} \). Then, same as the case of \( \frac{1}{2n^*} < \frac{v-p^*_t}{t} \) under \( 0 \leq p^*_o - p^*_t \leq e \), \( \Pi^* \) is linearly increasing in \( p_o \), and hence we can increase \( \Pi^* \) by increasing \( p_o \), which is a contradiction. Suppose \( \frac{v-p^*_t-e}{t} < \frac{1}{2n^*} < \frac{v-p^*_t}{t} \).

Then, since \( p^*_o - p^*_t > e \), we have \( \frac{1}{2n^*} = \frac{v-p^*_t-e}{t} \). Thus, the optimal profit is

\[
\Pi^* = (p^*_o - c)\alpha \mu \left( \frac{v-p^*_t}{t} \right) + (p^*_o - c)2(1-\alpha) \mu \left( \frac{v-p^*_t}{t} \right) - ch(z^*(p^*_t))\gamma \left( 2(1-\alpha) \frac{v-p^*_t}{t} - s \right) n^*. 
\]

Since \( \Pi^* \) is a linear function of \( n^* \), we can increase \( \Pi^* \) by either increasing or decreasing \( n \), hence contradiction. Suppose \( \frac{1}{2n^*} > \frac{v-p^*_t}{t} \). Then, same as the case of \( \frac{1}{2n^*} > \frac{v-p^*_t-e}{t} \) under \( 0 \leq p^*_o - p^*_t \leq e \), \( \Pi^* \) is linearly increasing in \( n \), and hence we can increase \( \Pi^* \) by increasing \( n \), which is contradiction. Therefore, if \( p^*_o - p^*_t > e \), we must have either \( \frac{1}{2n^*} = \frac{v-p^*_t}{t} \) or \( \frac{1}{2n^*} = \frac{v-p^*_t-e}{t} \).

We have shown that the optimal strategy satisfies \( p^*_o \leq p^*_t \), and \( p^*_o = v - \frac{1}{2n^*} \) or \( p^*_o = v - \frac{v-p^*_t-e}{t} \). Then, based on the demand segmentation characterized in §4.1, we obtain that the optimal customer coverage is full circle if and only if \( p^*_o = p^*_t \).
Proof of Proposition 2 To begin with, it is easy to see that depending on the relationship between $\Pi'$ and $\Pi^*$, the optimal strategy is determined as stated in the proposition. As the case of $n^1 = 0$ is trivial, we focus on the case of $n^1 > 0$, that is, $\Pi^* \geq \Pi^1$ and $\Pi^1 \geq 0$.

First, we prove $p_i^1 \leq p_i^0 + \epsilon$ by contradiction. Suppose the opposite is true, that is, $p_i^1 - p_i^0 > \epsilon$. Then, we must have $p_i^0 < v - \epsilon$, because if $p_i^0 \geq v - \epsilon$, we would have $p_i^1 > p_i^0 + \epsilon \geq v$. Thus, the optimal demand segmentation is given by $d_o(n^1, p_o^0, p_i^1) = \alpha + \beta$ and $d_i(n^1, p_o^0, p_i^1) = 2(1-\alpha - \beta)\min\left(\frac{v - p_i^1}{t}, \frac{1}{2n^1}\right)$, and the optimal profit is

$$
\Pi' = (p_o^0 - c)(\alpha + \beta) + (p_i^1 - c)(1-\alpha - \beta)\mu \min\left(\frac{v - p_i^1}{t}, \frac{1}{2n^1}\right) 2n^1 - c\mu (z(p_i^1))^\sigma n^1 \sqrt{2(1-\alpha - \beta)\min\left(\frac{v - p_i^1}{t}, \frac{1}{2n^1}\right) - sn^1 - k}.
$$

Since $\Pi'$ is linearly increasing in $p_o$, we can increase $\Pi'$ by increasing $p_o$ by an infinitesimally small amount $\epsilon$ (which preserves the demand segmentation), hence contradiction. Therefore, we must have $p_i^1 \leq p_i^0 + \epsilon$ under the optimal strategy.

We next prove $\frac{1}{2n^1} < \frac{v - p_i^1}{t}$. First, suppose $\frac{1}{2n^1} < \frac{v - p_i^1}{t}$. Consider another strategy $(\tilde{n}, \tilde{p}_o, \tilde{p}_i) = (n^1, p_o^0 + \epsilon, p_i^1 + \epsilon)$, where $\epsilon$ is an infinitesimally small positive number. Since $\frac{1}{2n^1} < \frac{v - p_i^1}{t}$, the demand segmentation under this strategy is same as that under the optimal strategy, i.e., $d_o(\tilde{n}, \tilde{p}_o, \tilde{p}_i) = d_o(n^1, p_o^1, p_i^1)$ and $d_i(\tilde{n}, \tilde{p}_o, \tilde{p}_i) = d_i(n^1, p_o^1, p_i^1)$. The optimal profit is

$$
\Pi' = (p_o^0 - c)d_o(n^1, p_o^1, p_i^1)\mu + (p_i^1 - c)d_i(n^1, p_o^1, p_i^1)\mu n^1 - c\mu (z(p_i^1))^\sigma n^1 \sqrt{d_i(n^1, p_o^1, p_i^1)} - sn^1 - k,
$$

while the profit under strategy $(\tilde{n}, \tilde{p}_o, \tilde{p}_i)$ is

$$
\Pi(\tilde{n}, \tilde{p}_o, \tilde{p}_i) = (p_o^0 + \epsilon - c)d_o(n^1, p_o^1, p_i^1)\mu + (p_i^1 + \epsilon - c)d_i(n^1, p_o^1, p_i^1)\mu n^1 - c\mu (z(p_i^1 + \epsilon))^\sigma n^1 \sqrt{d_i(n^1, p_o^1, p_i^1)} - sn^1 - k.
$$

Thus,

$$
\Pi(\tilde{n}, \tilde{p}_o, \tilde{p}_i) - \Pi' = \epsilon d_o(n^1, p_o^1, p_i^1)\mu + \left[(p_i^1 + \epsilon - c)d_i(n^1, p_o^1, p_i^1)\mu n^1 - c\mu (z(p_i^1 + \epsilon))^\sigma n^1 \sqrt{d_i(n^1, p_o^1, p_i^1)}\right] - \left[(p_i^1 - c)d_i(n^1, p_o^1, p_i^1)\mu n^1 - c\mu (z(p_i^1))^\sigma n^1 \sqrt{d_i(n^1, p_o^1, p_i^1)}\right].
$$

Denote $\mu^1 = d_i(n^1, p_o^1, p_i^1)\mu n^1$ and $\sigma^1 = \sigma n^1 \sqrt{d_i(n^1, p_o^1, p_i^1)}$. Further, define

$$
\tilde{\Pi}_i(p_i) = (p_i - c)d_i(n^1, p_o^1, p_i^1)\mu n^1 - c\mu (z(p_i))^\sigma n^1 \sqrt{d_i(n^1, p_o^1, p_i^1)} = (p_i - c)\mu^1 - c\mu (z(p_i))^\sigma^1
$$
as a function of $p_i$. We next show that $\tilde{\Pi}_i(p_i) > 0$ at $p_i = p_i^1$, thus

$$
\Pi(\tilde{n}, \tilde{p}_o, \tilde{p}_i) - \Pi' = \epsilon d_o(n^1, p_o^1, p_i^1)\mu + \tilde{\Pi}_i(p_i^1 + \epsilon) - \tilde{\Pi}_i(p_i^1) > \epsilon d_o(n^1, p_o^1, p_i^1)\mu > 0,
$$

leading to a contradiction.

We can write $\tilde{\Pi}_i(p_i)$ equivalently as

$$
\tilde{\Pi}_i(p_i) = (p_i - c)\mu^1 - p_i \phi(z(p_i))^\sigma^1.
$$
To show that \( \tilde{\Pi}'(p_i) > 0 \) at \( p_i = p_i^1 \), first notice that \( \tilde{\Pi}(p_i^1) > 0 \) because if \( \tilde{\Pi}(p_i) \leq 0 \), we can increase \( \Pi^1 \) by setting \( n = 0 \). \( \tilde{\Pi}(p_i^1) > 0 \) implies

\[
\mu^1 > \frac{p_i^1 \phi(z^1(p_i^1))}{p_i^1 - c} \cdot \sigma^1.
\]

Then, taking derivative of \( \tilde{\Pi}_i(p_i) \) yields

\[
\tilde{\Pi}'_i(p_i)|_{p_i = p_i^1} = \mu^1 - \left[ \phi(z^1(p_i^1)) - \frac{z^1(p_i^1)c}{p_i^1} \right] \sigma^1
\]

\[
> \sigma^1 \left[ \frac{p_i^1 \phi(z^1(p_i^1))}{p_i^1 - c} - \phi(z^1(p_i^1)) + \frac{z^1(p_i^1)c}{p_i^1} \right]
\]

\[
= \frac{\epsilon \sigma^1}{p_i^1 - c} \left[ \phi(z^1(p_i^1)) + \frac{z^1(p_i^1)(p_i^1 - c)}{p_i^1} \right].
\]

Let

\[
g(p_i) = \phi(z^1(p_i)) + \frac{z^1(p_i)(p_i - c)}{p_i}.
\]

The derivative of \( g(p_i) \) is

\[
g'(p_i) = \frac{c(p_i - c)}{p_i^1 \phi(z^1(p_i))} \geq 0.
\]

Then, \( g(p_i^1) \geq 0 \) is guaranteed by \( g(c) = 0 \). Note that \( \phi(z^1(c)) = 0 \). Thus, to show \( g(c) = 0 \), it suffices to show

\[
\lim_{p_i \to c} z^1(p_i)(p_i - c) = 0.
\]

By L’Hospital’s Rule,

\[
\lim_{p_i \to c} \frac{p_i - c}{z^1(p_i)} = \lim_{p_i \to c} \frac{1}{z′(p_i) \phi(z^1(p_i))} = \lim_{p_i \to c} \frac{p_i^2[z^1(p_i)]^2 \phi(z^1(p_i))}{-c} = 0,
\]

where \( \lim_{p_i \to c} [z^1(p_i)]^2 \phi(z^1(p_i)) = 0 \) because \( \phi(z^1(p_i)) \) has exponential decay in \( z^1(p_i) \). Thus, we have \( g(p_i^1) \geq 0 \), and hence \( \tilde{\Pi}'_i(p_i) > 0 \) at \( p_i = p_i^1 \).

We have shown that \( \frac{1}{2\sigma} > \frac{-p_i^1}{t} \) does not occur in the optimal strategy. Second, suppose \( \frac{1}{2\sigma} > \frac{-p_i^1}{t} \). In this case, we have \( \frac{p_i^1}{t} - \frac{p_i^2}{t} \leq \frac{v-p_i^1}{t} < \frac{1}{2\sigma} \). Moreover, if \( p_i^1 < v - \epsilon \), we have \( \frac{p_i^1}{t} - \frac{p_i^1 + \epsilon}{t} < \frac{v-p_i^1}{t} < \frac{1}{2\sigma} \). Thus, \( d_i(n^1, p_i^1, p_i^2) \) reduces to \( d_i(p_i^2, p_i^1) \) which is independent of \( n^1 \), and hence \( \Pi^1 \) is linear in \( n \). Then, we can increase \( \Pi^1 \) by either increasing or decreasing \( n \), hence contradiction. Note that since decreasing \( n \) will lead to \( n = 0 \) which is not optimal, we can only increase \( n \) in this case. Therefore, we must have \( \frac{1}{2\sigma} = \frac{v-p_i^1}{t} \), or \( p_i^1 = v - \frac{1}{2\sigma} \). Finally, since \( p_i^1 \leq p_i^2 + \epsilon \) and \( \frac{1}{2\sigma} = \frac{v-p_i^1}{t} \), based on the demand segmentation characterized in §4.2, we obtain that the optimal customer coverage is always full circle. □

Proof of Proposition 3 Proposition B.3 compares Case 1 and Case 2 for \( \sigma = 0 \). In Case 1, the optimal strategy is achieved in Subcase 1.2 (\( 0 \leq p_i - p_o \leq \epsilon \)), in which case the profit function is continuous in all decision variables as well as \( \sigma \). Similarly, in Case 2, the optimal strategy is achieved either in Subcase 2.1 (\( p_i - p_o < 0 \) and \( p_o \geq v - \epsilon \)) or in Subcase 2.2 (\( p_i - p_o < 0 \) and \( p_o < v - \epsilon \)). In either case, the profit function is continuous in all decision variables as well as \( \sigma \). Therefore, when \( \sigma \) is sufficiently small, the results follow from Proposition B.3 by continuity. □

Proof of Proposition 4 We focus on \( \Pi^1 \geq 0 \), because the case of \( \Pi^1 < 0 \) is trivial. We first show that the optimal strategy is not achieved in the subcase of \( p_i - p_o < 0 \). Suppose \( p_i^1 > p_i^2 \). Then, the optimal demand segmentation is given by \( d_i(p_o^1, p_i^2) = 0 \) and \( d_i(p_o^2, p_i^1) = 1 \), and the optimal profit is

\[
\Pi^1 = (p_i^1 - c)\mu - c_1 h(z^1(p_i^1))\sigma - k - k'\).
\]
Now, consider another strategy \((\tilde{p}_o, \tilde{p}_i) = (p^1_o, p^1_i)\). Under this strategy, the demand segmentation is given by 
\(d_o(\tilde{p}_o, \tilde{p}_i) = \alpha\) and \(d_i(\tilde{p}_o, \tilde{p}_i) = 1 - \alpha\), and the profit is

\[\Pi(\tilde{p}_o, \tilde{p}_i) = (p^1_i - c)\mu - c, h(z^1(p^1_i))\sigma\sqrt{1 - \alpha - k - k'} .\]

It is easy to see that \(\Pi(\tilde{p}_o, \tilde{p}_i) > \Pi^i\), hence contradiction. Therefore, we must have \(p^i_o \leq p^1_i\) under the optimal strategy.

Consider the subcase of \(p_i - p_o \geq e\). In this case, the demand segmentation is given by 
\(d_o(p_o, p_i) = \alpha + \beta\) and \(d_i(p_o, p_i) = 1 - \alpha - \beta\), and the profit function is

\[\Pi(p_o, p_i) = (p_i - c)(\alpha + \beta)\mu + (p_i - c)(1 - \alpha - \beta)\mu - c, h(z^1(p_i))\sigma\sqrt{1 - \alpha - \beta - k - k'} .\]

Since \(\Pi(p_o, p_i)\) is increasing in \(p_o\), the optimal \(p_o\) is \(p^i_o(p_i) = p_i - e\), which reduces the profit function to

\[\Pi(p_i) = (p_i - c)\mu - e(\alpha + \beta)\mu - c, h(z^1(p_i))\sigma\sqrt{1 - \alpha - \beta - k - k'} .\]

Note that we can equivalently write \(\Pi(p_i)\) as

\[\Pi(p_i) = (p_i - c)\mu - e(\alpha + \beta)\mu - c, h(z^1(p_i))\sigma\sqrt{1 - \alpha - \beta - k - k'},\]

Taking derivatives yields

\[\Pi'(p_i) = \mu - \frac{\phi(z^1(p_i)) - \frac{z^1(p_i)c^z}{p_i - c}}{p_i} \sigma\sqrt{1 - \alpha - \beta} ,\]

\[\Pi''(p_i) = -\frac{c}{(p_i - c)^2}\phi(z^1(p_i)) \cdot \sigma\sqrt{1 - \alpha - \beta} > 0.\]

Thus, \(\Pi(p_i)\) is convex in \(p_i\). Since \(p_i = c\) results in a negative profit, the optimal \(p_i\) in this subcase is \(p_i^1 = v\); correspondingly, \(p_i^1 = v - e\), and the resulting profit is

\[(v - c)\mu - e(\alpha + \beta)\mu - c, h(z^1(v))\sigma\sqrt{1 - \alpha - \beta - k - k'} .\]  \hspace{1cm} (A.1)

Next, consider the subcase of \(0 \leq p_i - p_o < e\). In this case, the demand segmentation is given by \(d_o(p_o, p_i) = \alpha\) and \(d_i(p_o, p_i) = 1 - \alpha\), and the profit function is

\[\Pi(p_o, p_i) = (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu - c, h(z^1(p_i))\sigma\sqrt{1 - \alpha - k - k'} .\]

Following the same analysis as in the subcase of \(p_i - p_o \geq e\), we obtain that the optimal solution in this subcase is \(p_i^1 = p_i^o = v\), and the resulting profit is

\[(v - c)\mu - c, h(z^1(v))\sigma\sqrt{1 - \alpha - k - k'} .\]  \hspace{1cm} (A.2)

Therefore, combining the local optimums from the two subcases, we obtain that the optimal strategy is \(p_i^1 = v\), and \(p_i^o = v\) or \(v - e\). Moreover, by comparing (A.1) and (A.2), we obtain that \(p_i^o = v\) if

\[e > \frac{c, h(z^1(v))\sigma(\sqrt{1 - \alpha - \sqrt{1 - \alpha - \beta})}{(\alpha + \beta)\mu} .\]

and \(p_i^o = v - e\) otherwise. Finally, based on the demand segmentation characterized in §4.3, we obtain that the optimal customer coverage is always full circle. \(\square\)
Proof of Proposition 5 Proposition B.5 compares Case 3 to the other two cases for \( \sigma = 0 \). In Case 3, the optimal strategy is achieved in Subcase 3.2 \((0 \leq p_i - p_o < e)\), in which case the profit function is continuous in all decision variables as well as \( \sigma \). Therefore, when \( \sigma \) is sufficiently small, the results follow from Proposition B.5 by continuity. \( \square \)

Proof of Proposition 6 Under the cost-sharing contract, for any \((w, \beta)\), the supplier’s optimal inventory order quantity is \( q_S^* = d^1 \mu + z_S^* \sigma \sqrt{d^1} \) where

\[
  z_S^* = \Phi^{-1} \left( 1 - \frac{(1 - \beta)c_r}{w} \right).
\] (A.3)

The decentralized supply chain can be coordinated if \( z_S^* = z^* \). From (8) and (A.3), we obtain that \( z_S^* = z^* \) requires \( \frac{(1-\beta)c_r}{w} = \frac{w}{v - c_p} \) which is equivalent to (10). Then, replacing \( w \) by \( (1 - \beta)(v - c_p) \) in \( \Pi_S(q, w, \beta) \), we have \( \Pi_S(q_S^*, w, \beta) = (1 - \beta) \cdot \Pi(q^*) \). \( \square \)
Appendix B: Analysis for $\sigma = 0$ and Proofs

In this section, we analyze the special case with $\sigma = 0$. With $\sigma = 0$, we are able to obtain closed-form solutions for the optimal strategies under each production system. The closed-form solutions enable us to obtain sharper insights by comparing the optimal strategies and profits between different cases. The analysis and results in this section also assist us in proving some of the results in the main text for small $\sigma$.

To avoid triviality, we focus on the scenario when both channels are used. The results and proofs are presented below. Proposition B.1 characterizes the optimal strategy and profit under Case 1. Proposition B.2 characterizes the optimal strategy and profit under Case 2. Proposition B.3 compares Case 1 and Case 2. Proposition B.4 characterizes the optimal strategy and profit under Case 3. Proposition B.5 compares Case 3 to the previous two cases.

**Proposition B.1.** Under the traditional system, the following results hold when $\sigma = 0$:

(i) The firm offers $n^* = \sqrt{\frac{\mu}{2s}}$ types of product at optimal prices $p_{o}^* = p_{i}^* = v - \sqrt{\frac{s}{2\mu}}$.

(ii) All Type I customers purchase online, all Type II and Type III customers purchase in-store. Thus, the firm’s online demand is $\alpha \mu$ and the in-store demand is $(1 - \alpha) \mu$.

(iii) The firm’s optimal profit is $\Pi^* = (v - c) \mu - \sqrt{2s} \mu$.

**Proposition B.2.** When the firm uses 3D printing online, the following results hold when $\sigma = 0$:

(i) There exists a threshold $\bar{e} = \sqrt{\frac{st}{2} \frac{\sqrt{st}}{2\mu} + \sqrt{2} \left(1 - \frac{\alpha}{4}\right) \mu - \frac{2s}{\alpha + \beta}}$ such that if $e \geq \bar{e}$, the firm offers $n^1 = \sqrt{\left(1 - \frac{\alpha}{4}\right) \mu^2 / 2s}$ types of product at optimal prices

$$p_{o}^1 = v - \frac{1}{2} \sqrt{\frac{st}{2 \left(1 - \frac{\alpha}{4}\right) \mu}}$$

and

$$p_{i}^1 = v - \sqrt{\frac{st}{2 \left(1 - \frac{\alpha}{4}\right) \mu}}.$$ $p_{o}^1 > p_{i}^1$. Under the optimal product offering and pricing strategy, Type I customers purchase online if $\frac{v - p_{o}^1}{\frac{1}{2}} \leq x \leq \frac{v - p_{i}^1}{\frac{1}{4}}$ and in-store if $0 \leq x < \frac{v - p_{i}^1}{\frac{1}{4}}$, all Type II and Type III customers purchase in-store. Thus, the firm’s online demand is $\frac{\alpha}{2} \mu$ and the in-store demand is $(1 - \frac{\alpha}{2}) \mu$. The firm’s optimal profit is

$$\Pi^1 = (v - c) \mu - \sqrt{2 \left(1 - \frac{\alpha}{4}\right) st \mu - k}.$$ $\Pi^1 > \Pi^*$. Under the optimal product offering and pricing strategy, Type I customers purchase online if $\frac{v - p_{o}^1}{\frac{1}{2}} \leq x \leq \frac{v - p_{i}^1}{\frac{1}{4}}$ and in-store if $0 \leq x < \frac{v - p_{i}^1}{\frac{1}{4}}$, all Type II and Type III customers purchase in-store. Thus, the firm’s online demand is $\frac{\alpha}{2} \mu$ and the in-store demand is $(1 - \frac{\alpha}{2}) \mu$. The firm’s optimal profit is

(ii) If $e < \bar{e}$, the firm offers $n^1 = \sqrt{\frac{(1 - \frac{\alpha}{4}) \mu^2}{2s - \frac{\beta^2 e^2}{(\alpha + \beta)^2}}}$. $p_{o}^1$, $p_{i}^1$, $p_{o}^1 > p_{i}^1$. Under the optimal product offering and pricing strategy, Type I customers purchase online if $\frac{v - p_{o}^1}{\frac{1}{2}} \leq x \leq \frac{v - p_{i}^1}{\frac{1}{4}}$ and in-store if $0 \leq x < \frac{v - p_{i}^1}{\frac{1}{4}}$, all Type II and Type III customers purchase in-store. Thus, the firm’s online demand is $\frac{\alpha}{2} \mu$ and the in-store demand is $(1 - \frac{\alpha}{2}) \mu$. The firm’s optimal profit is

$$\Pi^1 = (v - c) \mu - \sqrt{2 \left(1 - \frac{\alpha}{4}\right) st \mu - k}.$$
Moreover, \( p^*_o > p^*_i \). Under the optimal product offering and pricing strategy, Type I customers purchase online if \( \frac{p^*_o - p^*_i}{t} \leq x \leq \frac{1}{2n^*} \), and in-store if \( 0 \leq x < \frac{p^*_o - p^*_i}{t} \), Type II customers purchase online if \( \frac{p^*_o - p^*_i + e}{t} \leq x \leq \frac{1}{2n^*} \), and in-store if \( 0 \leq x < \frac{p^*_o - p^*_i + e}{t} \), and all Type III customers purchase in-store. Thus, the firm’s online demand is

\[
\left[ \frac{\alpha + \beta}{2} - \beta e \sqrt{\frac{(1 - \frac{\alpha + \beta}{4}) \mu}{2st - \frac{\beta^2 \mu e^2}{\alpha + \beta}}} \right] \mu.
\]

and the in-store demand is

\[
\left[ 1 - \frac{\alpha + \beta}{2} + \beta e \sqrt{\frac{(1 - \frac{\alpha + \beta}{4}) \mu}{2st - \frac{\beta^2 \mu e^2}{\alpha + \beta}}} \right] \mu.
\]

The firm’s optimal profit is

\[
\Pi^* = (v - c)\mu - k.
\]

**Proposition B.3.** The following results hold when \( \sigma = 0 \):

(i) \( n^* \leq n^* \).

(ii) \( p^*_o > p^*_o \).

(iii) \( p^*_i < p^*_i \).

(iv) \( \alpha^* < \alpha^* \).

(v) Compared to the traditional system, if \( e > \xi \) where

\[
\xi = \frac{(\beta - \alpha)^+}{\beta} \sqrt{\frac{(\alpha + \beta)st}{2(\alpha + \beta - \alpha\beta)\mu}},
\]

the firm’s online demand is lower and the in-store demand is higher when it uses 3D printing online. If \( e \leq \xi \), the opposite occurs. Moreover, \( \xi < \bar{\xi} \).

(vi) \( \Pi^* \geq \Pi^* \) if and only if \( k \leq \bar{k} \) where

\[
k = \begin{cases} 
\sqrt{2st\mu} - \sqrt{2 \left( 1 - \frac{\alpha}{4} \right) s t \mu} & \text{if } e \geq \bar{e}, \\
\sqrt{2st\mu} - \sqrt{\frac{1 - \alpha + \beta}{4} \mu - \frac{\beta \mu e^2}{2}} & \text{if } e < \bar{e}.
\end{cases}
\]

Moreover, \( \bar{k} > 0 \), so \( \Pi^* > \Pi^* \) when \( k = 0 \).

(vii) \( \Pi^* - \Pi^* \) is increasing in both \( t \) and \( s \).

**Proposition B.4.** When the firm uses 3D printing in-store as well, the following results hold when \( \sigma = 0 \):

(i) The firm offers infinite types of product at optimal prices \( p^*_o = p^*_i = v \).

(ii) All Type I customers purchase online, all Type II and Type III customers purchase in-store. Thus, the firm’s online demand is \( \alpha \mu \) and the in-store demand is \( (1 - \alpha)\mu \).

(iii) The firm’s optimal profit is \( \Pi^* = (v - c)\mu - k - k' \).

**Proposition B.5.** The following results hold when \( \sigma = 0 \):

(i) \( p^*_o > p^*_o > p^*_o > p^*_i > p^*_i > p^*_i \).

(ii) \( z^* > z^* > z^* \).
(iii) Compared to the case where the firm only adopts 3D printing online, if \( e > \bar{e} \), the firm’s online demand is higher and the in-store demand is lower when it adopts 3D printing in-store as well. If \( e \leq \bar{e} \), the opposite occurs.

(iv) \( \Pi_1^1 \geq \Pi_1^* \) if and only if \( k' \leq \bar{k} \) where

\[
\bar{k}' = \begin{cases} 
\sqrt{2 \left(1 - \frac{\alpha}{4}\right) s t \mu} & \text{if } e \geq \bar{e}, \\
\sqrt{\left(1 - \frac{\alpha + \beta}{4}\right) \left(2 s t - \frac{\beta^2 \mu t^2}{\alpha + \beta}\right) \mu + \frac{\beta \mu t^2}{2}} & \text{if } e < \bar{e}.
\end{cases}
\]

Moreover, \( \bar{k}' > 0 \), so \( \Pi_1^1 > \Pi_1^* \) when \( k' = 0 \).

(v) \( \Pi_1^* - \Pi_1^1 \) and \( \Pi_1^1 - \Pi_1^* \) are increasing in both \( t \) and \( s \).

Proof of Proposition B.1  When \( \sigma = 0 \), based on the demand segmentation in §4.1, we derive the firm’s profit function as follows:

\[
\Pi(n, p_o, p_t) = \begin{cases} 
(p_o - c) \left( \alpha \mu \min \left( \frac{v - p_o}{t}, \frac{1}{2n} \right) + \beta \mu \min \left( \frac{v - p_o - e}{t}, \frac{1}{2n} \right) \right) 2n & \text{if } p_t - p_o > e, \\
(p_o - c) (1 - \alpha - \beta) \mu \min \left( \frac{v - p_t}{t}, \frac{1}{2n} \right) 2n - s_n & \text{if } 0 \leq p_t - p_o \leq e, \\
(p_o - c) \mu \min \left( \frac{v - p_t}{t}, \frac{1}{2n} \right) 2n - s_n & \text{if } p_t - p_o < 0.
\end{cases}
\]

To analyze the profit function and derive the optimal strategy, we define three subcases in Case 1: Case 1.1 \( (p_t - p_o > e) \), Case 1.2 \( (0 \leq p_t - p_o \leq e) \), Case 1.3 \( (p_t - p_o < 0) \).

In Case 1.1, depending on \( n \), the profit function becomes

\[
\Pi_{1.1}(n, p_o, p_t) = \begin{cases} 
(p_o - c) (\alpha + \beta) \mu + (p_t - c) (1 - \alpha - \beta) \mu - s_n & \text{if } \frac{1}{2n} < \frac{v - p_t}{t}, \\
(p_o - c) (\alpha + \beta) \mu + (p_t - c) (1 - \alpha - \beta) \mu \left( \frac{v - p_t}{t} \right) 2n - s_n & \text{if } \frac{v - p_t}{t} \leq \frac{1}{2n} < \frac{v - p_o - e}{t}, \\
(p_o - c) \left( \alpha \mu + \beta \mu \left( \frac{v - p_o - e}{t} \right) 2n \right) + (p_t - c) (1 - \alpha - \beta) \mu \left( \frac{v - p_t}{t} \right) 2n - s_n & \text{if } \frac{v - p_o - e}{t} \leq \frac{1}{2n} \leq \frac{v - p_o}{t}, \\
(p_o - c) \left( \alpha \mu \left( \frac{v - p_o}{t} \right) + \beta \mu \left( \frac{v - p_o - e}{t} \right) \right) 2n + (p_t - c) (1 - \alpha - \beta) \mu \left( \frac{v - p_t}{t} \right) 2n - s_n & \text{if } \frac{1}{2n} \geq \frac{v - p_o}{t}.
\end{cases}
\]
If \( \frac{1}{2n} < \frac{v-p_\alpha}{t} \) which requires \( n \) is large enough, \( \Pi_{1.1}(n, p_o, p_i) \) is decreasing in \( n \), so \( \frac{1}{2n} < \frac{v-p_\alpha}{t} \) is dominated by \( \frac{v-p_\alpha}{t} < \frac{v-p_o}{t} \) which requires \( p_o \) is small enough, \( \Pi_{1.1}(n, p_o, p_i) \) is increasing in \( p_o \), so \( \frac{v-p_\alpha}{t} < \frac{v-p_o}{t} \) is dominated by \( \frac{1}{2n} \geq \frac{v-p_o}{t} \). Thus, the optimal strategy can only be supported by \( \frac{1}{2n} \geq \frac{v-p_o}{t} \) in Case 1.1.

In Case 1.2, depending on \( n \), the profit function becomes

\[
\Pi_{1.2}(n, p_o, p_i) = \begin{cases} (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu - sn & \text{if } \frac{1}{2n} < \frac{v-p_\alpha}{t}, \\ (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu \left( \frac{v-p_\alpha}{t} \right) 2n - sn & \text{if } \frac{v-p_\alpha}{t} < \frac{1}{2n} < \frac{v-p_o}{t}, \\ (p_o - c)\alpha\mu \left( \frac{v-p_o}{t} \right) 2n + (p_i - c)(1 - \alpha)\mu \left( \frac{v-p_i}{t} \right) 2n - sn & \text{if } \frac{1}{2n} \geq \frac{v-p_o}{t}. \end{cases}
\]

If \( \frac{1}{2n} < \frac{v-p_\alpha}{t} \) which requires \( n \) is large enough, \( \Pi_{1.2}(n, p_o, p_i) \) is decreasing in \( n \), so the optimal strategy can only be supported by \( \frac{1}{2n} \geq \frac{v-p_\alpha}{t} \) in Case 1.3. Moreover, for any \(( n, p_o, p_i )\) in Case 1.3 such that \( \frac{1}{2n} \geq \frac{v-p_\alpha}{t} \), we can pick \(( n, \hat{p}_o, \hat{p}_i ) = ( n, p_o, p_i ) \) which is in Case 1.2 and yields \( \Pi_{1.2}(n, \hat{p}_o, \hat{p}_i) = \Pi_{1.3}(n, p_o, p_i) \). Thus, Case 1.3 is dominated by Case 1.2, and hence the optimal strategy can only be supported by Case 1.1 with \( \frac{1}{2n} \geq \frac{v-p_o}{t} \) or Case 1.2 with \( \frac{1}{2n} \geq \frac{v-p_o}{t} \).

We now optimize \( n \). In Case 1.1, \( \Pi_{1.1}(n, p_o, p_i) \) is linear in \( n \) for \( \frac{1}{2n} \geq \frac{v-p_\alpha}{t} \) and for \( \frac{v-p_o}{t} < \frac{1}{2n} < \frac{v-p_\alpha}{t} \), so the optimal \( n \) is either \( n^*(p_o, p_i) = \frac{t}{2(v-p_o)} \) or \( n^*(p_o, p_i) = \frac{t}{2(v-p_o)} \). With \( n^*(p_o, p_i) = \frac{t}{2(v-p_o)} \), the profit function reduces to

\[
\Pi_{1.1.1}(p_o, p_i) = (p_o - c) \left[ \alpha\mu + \beta \mu \left( \frac{v-p_o - e}{v-p_o} \right) \right] + (p_i - c)(1 - \alpha - \beta)\mu \left( \frac{v-p_i}{v-p_o} \right) - \frac{st}{2(v-p_o)}. \quad (B.1)
\]

With \( n^*(p_o, p_i) = \frac{t}{2(v-p_o)} \), the profit function reduces to

\[
\Pi_{1.1.2}(p_o, p_i) = (p_o - c)(\alpha + \beta)\mu + (p_i - c)(1 - \alpha - \beta)\mu \left( \frac{v-p_i}{v-p_o} - \frac{e}{2(v-p_o)} \right) - \frac{st}{2(v-p_o)}. \quad (B.2)
\]

In Case 1.2, \( \Pi_{1.2}(n, p_o, p_i) \) is linear in \( n \) for \( \frac{1}{2n} \geq \frac{v-p_\alpha}{t} \), so the optimal \( n \) is \( n^*(p_o, p_i) = \frac{t}{2(v-p_o)} \) which reduces the profit function to

\[
\Pi_{1.2}(p_o, p_i) = (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu \left( \frac{v-p_i}{v-p_o} \right) - \frac{st}{2(v-p_o)}. \quad (B.3)
\]

Next, we optimize \( p_i \). We only need to consider \( \Pi_{1.1.1}(p_o, p_i) \), \( \Pi_{1.1.2}(p_o, p_i) \) and \( \Pi_{1.2}(p_o, p_i) \). First, consider \( \Pi_{1.1.1}(p_o, p_i) \). Taking derivative of (B.1) with respect to \( p_i \) yields

\[
\frac{\partial \Pi_{1.1.1}}{\partial p_i} = (1 - \alpha - \beta)\mu \left( \frac{v + c - 2p_i}{v - p_o} \right). 
\]
Thus, \( \Pi_{1.1.1}(p_o, p_i) \) is concave in \( p_i \) and solving the first-order condition yields \( p_i = \frac{v + c}{2} \). Since Case 1.1 requires \( p_i - p_o > e \), the optimal \( p_i \) is

\[
p_i^*(p_o) = \begin{cases} 
\frac{v + c}{2} & \text{for } p_o < \frac{v + c}{2} - e, \\
p_o + e & \text{for } p_o \geq \frac{v + c}{2} - e.
\end{cases}
\]

With \( p_i = p_i^*(p_o) \), \( \Pi_{1.1.1}(p_o, p_i) \) reduces to

\[
\Pi_{1.1.1}(p_o) = \begin{cases} 
(p_o - c)\left[\alpha \mu + \beta \mu \left(\frac{v - p_o - e}{v - p_o}\right)\right] + (1 - \alpha - \beta)\mu \left(\frac{v - c}{2}\right)^2 \frac{1}{v - p_o} - \frac{st}{2(v - p_o)} & \text{for } p_o < \frac{v + c}{2} - e, \\
(p_o - e)\left[\alpha \mu + \beta \mu \left(\frac{v - p_o - e}{v - p_o}\right)\right] + (p_o + e - c)(1 - \alpha - \beta)\mu \left(\frac{v - p_o - e}{v - p_o}\right) & \text{for } p_o \geq \frac{v + c}{2} - e.
\end{cases}
\]

Second, consider \( \Pi_{1.2.1}(p_o, p_i) \). Following similar analysis for \( \Pi_{1.1.1}(p_o, p_i) \), we can obtain from (B.2) that for \( \Pi_{1.1.2}(p_o, p_i) \), the optimal \( p_i \) is same as \( p_i^*(p_o) \) for \( \Pi_{1.1.1}(p_o, p_i) \), and \( \Pi_{1.1.2}(p_o, p_i) \) is reduced to

\[
\Pi_{1.1.2}(p_o) = \begin{cases} 
(p_o - c)\left(\alpha + \beta\right)\mu + (1 - \alpha - \beta)\mu \left(\frac{v - c}{2}\right)^2 \frac{1}{v - p_o} - \frac{st}{2(v - p_o)} & \text{for } p_o < \frac{v + c}{2} - e, \\
(p_o - c)\left(\alpha + \beta\right)\mu + (p_o + e - c)(1 - \alpha - \beta)\mu \left(\frac{v - p_o - e}{v - p_o}\right) & \text{for } p_o \geq \frac{v + c}{2} - e.
\end{cases}
\]

Third, consider \( \Pi_{1.2}(p_o, p_i) \). From (B.3) it is easy to see that \( \Pi_{1.2}(p_o, p_i) \) is concave in \( p_i \) and the first-order condition yields \( p_i = \frac{v + c}{2} \). Since Case 1.2 requires \( 0 \leq p_i - p_o \leq e \), the optimal \( p_i \) is

\[
p_i^*(p_o) = \begin{cases} 
p_o + e & \text{for } p_o < \frac{v + c}{2} - e, \\
\frac{v + c}{2} & \text{for } \frac{v + c}{2} - e \leq p_o < \frac{v + c}{2}, \\
p_o & \text{for } p_o \geq \frac{v + c}{2}.
\end{cases}
\]

With \( p_i = p_i^*(p_o) \), \( \Pi_{1.2}(p_o, p_i) \) reduces to

\[
\Pi_{1.2}(p_o) = \begin{cases} 
(p_o - c)\alpha \mu + (p_o + e - c)(1 - \alpha)\mu \left(\frac{v - p_o - e}{v - p_o}\right) - \frac{st}{2(v - p_o)} & \text{for } p_o < \frac{v + c}{2} - e, \\
(p_o - c)\alpha \mu + (1 - \alpha)\mu \left(\frac{v - c}{2}\right)^2 \frac{1}{v - p_o} - \frac{st}{2(v - p_o)} & \text{for } \frac{v + c}{2} - e \leq p_o < \frac{v + c}{2}, \\
(p_o - c)\mu - \frac{st}{2(v - p_o)} & \text{for } p_o \geq \frac{v + c}{2}.
\end{cases}
\]
Next, we optimize \( p_o \) and obtain \( \Pi_{1.1.1}^*, \Pi_{1.1.2}^* \) and \( \Pi_{1.2}^* \). By comparing \( \Pi_{1.1.1}^*, \Pi_{1.1.2}^* \) and \( \Pi_{1.2}^* \), we will obtain which subcase supports the optimal strategy. First, consider \( \Pi_{1.1.1}(p_o) \). For \( \frac{v+c}{2} - e \leq p_o < \frac{v+c}{2} \), from (B.5) we have
\[
\Pi_{1.1.1}(p_o) < (p_o - c)\alpha\mu + (p_o + e - c)(1 - \alpha)\mu \left( \frac{v - p_o - e}{v - p_o} \right) - \frac{st}{2(v - p_o)}
\leq (p_o - c)\alpha\mu + (1 - \alpha)\mu \left( \frac{v - c}{2} \right)^2 \frac{1}{v - p_o} - \frac{st}{2(v - p_o)}
= \Pi_{1.2}(p_o),
\]
where the first inequality is straightforward and the second inequality follows from the fact that \((p_o + e - c)(v - p_o - e)\) is maximized at \( p_o = \frac{v+c}{2} - e \) and its maximum value is \((\frac{v+c}{2})^2\). Moreover, for \( p_o \geq \frac{v+c}{2} \), from (B.5) we have
\[
\Pi_{1.1.1}(p_o) < (p_o - c)(\alpha + \beta)\mu + (p_o + e - c)(1 - \alpha - \beta)\mu \left( \frac{v - p_o - e}{v - p_o} \right) - \frac{st}{2(v - p_o)}
< (p_o - c)(\alpha + \beta)\mu + (p_o - c)(1 - \alpha - \beta)\mu - \frac{st}{2(v - p_o)}
= (p_o - c)\mu - \frac{st}{2(v - p_o)}
= \Pi_{1.2}(p_o),
\]
where the first inequality is straightforward and the second inequality follows from the fact that \((p_o + e - c)(v - p_o - e)\) is decreasing in \( w \) for \( p_o \geq \frac{v+c}{2} \) so \((p_o + e - c)(v - p_o - e) < (p_o - c)(v - p_o)\). Thus, for \( p_o \geq \frac{v+c}{2} - e \), \( \Pi_{1.1.1}(p_o) < \Pi_{1.2}(p_o) \). Now, consider \( p_o < \frac{v+c}{2} - e \). Taking derivative of (B.4) with respect to \( p_o \) yields
\[
\frac{\partial \Pi_{1.1.1}}{\partial p_o} = (\alpha + \beta)\mu + \frac{1}{(v - p_o)^2} \left[ -\beta\mu(v - c)e + (1 - \alpha - \beta)\mu \left( \frac{v - c}{2} \right)^2 - \frac{st}{2} \right].
\]
If \(-\beta\mu(v - c)e + (1 - \alpha - \beta)\mu \left( \frac{v - c}{2} \right)^2 - \frac{st}{2} \geq 0\), then \(\frac{\partial \Pi_{1.1.1}}{\partial p_o} > 0\) for \( p_o < \frac{v+c}{2} - e \), so for \( \Pi_{1.1.1}(p_o) \), the optimal \( p_o \) is achieved in \( p_o \geq \frac{v+c}{2} - e \), and hence we must have \( \Pi_{1.1.1}^* < \Pi_{1.2}^* \). If \(-\beta\mu(v - c)e + (1 - \alpha - \beta)\mu \left( \frac{v - c}{2} \right)^2 - \frac{st}{2} < 0\), then \(\frac{\partial \Pi_{1.1.1}}{\partial p_o} \) is decreasing in \( p_o \), so \( \Pi_{1.1.1}(p_o) \) is concave in \( p_o \) for \( p_o < \frac{v+c}{2} - e \). The derivative at \( p_o = \frac{v+c}{2} - e \) is
\[
\frac{\partial \Pi_{1.1.1}}{\partial p_o} \bigg|_{p_o=\frac{v+c}{2} - e} = \frac{1}{(\frac{v+c}{2} - e)^2} \left[ (\alpha + \beta)\mu \left( \frac{v - c}{2} + e \right)^2 - \beta\mu(v - c)e + (1 - \alpha - \beta)\mu \left( \frac{v - c}{2} \right)^2 - \frac{st}{2} \right]
\geq \frac{1}{(\frac{v+c}{2} - e)^2} \left[ \mu \left( \frac{v - c}{2} \right)^2 + \alpha\mu(v - c)e + (\alpha + \beta)\mu e^2 - \frac{st}{2} \right]
\geq 0,
\]
where the inequality follows from \((v - c)\mu - \sqrt{2st}\mu \geq 0\) which is \( \Pi^* \geq 0 \) when \( \sigma = 0 \). Since \( \Pi_{1.1.1}(p_o) \) is concave in \( p_o \) for \( p_o < \frac{v+c}{2} - e \), we then have \(\frac{\partial \Pi_{1.1.1}}{\partial p_o} > 0\) for \( p_o < \frac{v+c}{2} - e \), which again indicates \( \Pi_{1.1.1}^* < \Pi_{1.2}^* \).

Second, consider \( \Pi_{1.1.2}(p_o) \). For \( p_o \geq \frac{v+c}{2} - e \), the derivative of (B.7) with respect to \( p_o \) is
\[
\frac{\partial \Pi_{1.1.2}}{\partial p_o} = \mu - \frac{st}{2(v - p_o - e)^2}.
\]
which is decreasing in \( p_o \), so \( \Pi_{1,1,2}(p_o) \) is concave in \( p_o \) for \( p_o \geq \frac{\nu + e}{2} - e \). At \( p_o = \frac{\nu + e}{2} - e \),

\[
\frac{\partial \Pi_{1,1,2}}{\partial p_o} \bigg|_{p_o=\frac{\nu + e}{2} - e} = \mu - \frac{2st}{(v-c)^2} \geq 0.
\]

Moreover, for \( p_o < \frac{\nu + e}{2} - e \), the derivative of (B.6) with respect to \( p_o \) is

\[
\frac{\partial \Pi_{1,1,2}}{\partial p_o} = (\alpha + \beta)\mu + \frac{1}{(v-p_o-e)^2} \left[ (1-\alpha-\beta)\mu \left( \frac{v-c}{2} \right)^2 - st \right].
\]

If \( (1-\alpha-\beta)\mu \left( \frac{v-c}{2} \right)^2 - \frac{st}{2} \geq 0 \), then \( \frac{\partial \Pi_{1,1,2}}{\partial p_o} > 0 \) for \( p_o < \frac{\nu + e}{2} - e \). If \( (1-\alpha-\beta)\mu \left( \frac{v-c}{2} \right)^2 - \frac{st}{2} < 0 \), then \( \frac{\partial \Pi_{1,1,2}}{\partial p_o} \) is decreasing in \( p_o \). Then, since

\[
\frac{\partial \Pi_{1,1,2}}{\partial p_o} \bigg|_{p_o=\frac{\nu + e}{2} - e} = \mu - \frac{2st}{(v-c)^2} \geq 0,
\]

we have \( \frac{\partial \Pi_{1,1,2}}{\partial p_o} > 0 \) for \( p_o < \frac{\nu + e}{2} - e \). Thus, we conclude that the optimal \( p_o \) is achieved in \( p_o \geq \frac{\nu + e}{2} - e \) and the first-order condition yields \( p_o^* = v - e - \sqrt{\frac{st}{2\nu}} \). Correspondingly, \( \Pi_{1,1,2} = (v-c)\mu - (\alpha + \beta)\mu e - \sqrt{2st\mu} \).

Third, consider \( \Pi_{1,2}(p_o) \). For \( p_o \geq \frac{\nu + e}{2} \), the derivative of (B.10) with respect to \( p_o \) is

\[
\frac{\partial \Pi_{1,2}}{\partial p_o} = \mu - \frac{st}{2(v-p_o)^2}
\]

which is decreasing in \( p_o \), so \( \Pi_{1,2}(p_o) \) is concave in \( p_o \) for \( p_o \geq \frac{\nu + e}{2} \). Moreover,

\[
\frac{\partial \Pi_{1,2}}{\partial p_o} \bigg|_{p_o=\frac{\nu + e}{2} - e} = \mu - \frac{2st}{(v-c)^2} \geq 0.
\]

For \( \frac{\nu + e}{2} - e \leq p_o < \frac{\nu + e}{2} \), the derivative of (B.9) with respect to \( p_o \) is

\[
\frac{\partial \Pi_{1,2}}{\partial p_o} = \alpha + \frac{1}{(v-p_o)^2} \left[ (1-\alpha)\mu \left( \frac{v-c}{2} \right)^2 - st \right].
\]

If \( (1-\alpha)\mu \left( \frac{v-c}{2} \right)^2 - \frac{st}{2} \geq 0 \), then \( \frac{\partial \Pi_{1,2}}{\partial p_o} > 0 \) for \( \frac{\nu + e}{2} - e \leq p_o < \frac{\nu + e}{2} \). If \( (1-\alpha)\mu \left( \frac{v-c}{2} \right)^2 - \frac{st}{2} < 0 \), then since

\[
\frac{\partial \Pi_{1,2}}{\partial p_o} \bigg|_{p_o=\frac{\nu + e}{2} - e} = \mu - \frac{2st}{(v-c)^2} \geq 0,
\]

we have \( \frac{\partial \Pi_{1,2}}{\partial p_o} > 0 \) for \( \frac{\nu + e}{2} - e \leq p_o < \frac{\nu + e}{2} \). Thus, \( \frac{\nu + e}{2} - e \leq p_o < \frac{\nu + e}{2} \) is dominated by \( p_o \geq \frac{\nu + e}{2} \). Finally, for \( p_o < \frac{\nu + e}{2} - e \), taking derivatives of (B.8) yields

\[
\frac{\partial \Pi_{1,1,2}}{\partial p_o} = \alpha + \frac{1}{(v-p_o)^3} \left[ (v-p_o-e)(v-p_o)-(p_o+e)c - \frac{st}{2(v-p_o)^2} \right],
\]

\[
\frac{\partial^2 \Pi_{1,1,2}}{\partial p_o^2} = -2(1-\alpha)\mu e(v-c+e) - \frac{st}{(v-p_o)^3} < 0.
\]

So, \( \Pi_{1,2}(p_o) \) is concave in \( p_o \) for \( p_o < \frac{\nu + e}{2} - e \). Moreover, at \( p_o = \frac{\nu + e}{2} - e \),

\[
\frac{\partial \Pi_{1,1,2}}{\partial p_o} \bigg|_{p_o=\frac{\nu + e}{2} - e} = \mu + \frac{1}{(\nu + e+e)^2} \left[ (1-\alpha)\mu \left( \frac{v-c}{2} \right)^2 - st \right] = \frac{\partial \Pi_{1,1,2}}{\partial p_o} \bigg|_{p_o=\frac{\nu + e}{2} - e} > 0.
\]

Note that we have shown that \( \frac{\partial \Pi_{1,2}}{\partial p_o} > 0 \) for \( \frac{\nu + e}{2} - e \leq p_o < \frac{\nu + e}{2} \). Therefore, we conclude that the optimal \( p_o \) is achieved in \( p_o \geq \frac{\nu + e}{2} \) and the first-order condition yields \( p_o^* = v - \sqrt{\frac{st}{2\nu}} \). Correspondingly, \( \Pi_{1,2} = (v-c)\mu - \sqrt{2st\mu} \). We have also shown that \( \Pi_{1,1,1} < \Pi_{1,2} \). Therefore, the optimal strategy is achieved in Case 1.2 and the optimal profit is \( \Pi = (v-c)\mu - \sqrt{2st\mu} \). Tracing back our analysis for Case 1.2, we obtain that \( p_o^* = v - \sqrt{\frac{st}{2\nu}}, \) \( p_o^* = p_o^* \), and \( n^* = \frac{\nu}{2(v-p_o^*)} = \frac{\nu^2}{2t} \). Moreover, under the optimal strategy, all Type I customers purchase online, all Type II and Type III customers purchase in-store. The proof is complete. □
Proof of Proposition B.2 When $\sigma = 0$, based on the demand segmentation in §4.2, we derive the firm’s profit function as follows:

$$
\Pi(n, p_o, p_i) = \begin{cases} 
(p_o - c)\alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right)^2n & 
\text{if } p_i - p_o < 0 \text{ and } p_o < v - e, \\
(p_o - c) \alpha \mu \min \left( \frac{p_o - p_i}{t}, \frac{1}{2n} \right) + (1 - \alpha)\mu \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) 2n - sn - k & 
\text{if } p_i - p_o > 0 \text{ and } p_o \geq v - e, \\
(p_o - c) \left[ \alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right) + \beta \mu \left( \frac{1}{2n} - \frac{p_o - p_i + e}{t} \right) \right] & 
\text{if } 0 \leq p_i - p_o \leq e \text{ and } p_o < v - e, \\
(p_o - c) \alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right) + (1 - \alpha)\mu \min \left( \frac{v - p_i}{t}, \frac{1}{2n} \right) 2n - sn - k & 
\text{if } p_i - p_o > e \text{ and } p_o < v - e.
\end{cases}
$$

To analyze the profit function and derive the optimal strategy, we define five subcases in Case 2: Case 2.1 ($p_i - p_o < 0$ and $p_o \geq v - e$), Case 2.2 ($p_i - p_o < 0$ and $p_o < v - e$), Case 2.3 ($p_i - p_o > 0$ and $p_o \geq v - e$), Case 2.4 ($0 \leq p_i - p_o \leq e$ and $p_o < v - e$), Case 2.5 ($p_i - p_o > e$ and $p_o < v - e$).

In Case 2.1, depending on $n$, the profit function becomes

$$
\Pi_{2.1}(n, p_o, p_i) = \begin{cases} 
(p_i - c)\mu - sn - k & 
\text{if } \frac{1}{2n} < \frac{p_o - p_i}{t}, \\
(p_o - c)\alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right) 2n + (p_i - c) \left[ \alpha \mu \left( \frac{p_o - p_i}{t} \right) 2n + (1 - \alpha)\mu \right] - sn - k & 
\text{if } \frac{p_o - p_i}{t} \leq \frac{1}{2n} < \frac{v - p_i}{t}, \\
(p_o - c)\alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right) 2n + (p_i - c) \left[ \alpha \mu \left( \frac{p_o - p_i}{t} \right) + (1 - \alpha)\mu \left( \frac{v - p_i}{t} \right) \right] 2n - sn - k & 
\text{if } \frac{1}{2n} \geq \frac{v - p_i}{t}.
\end{cases}
$$

If $\frac{1}{2n} < \frac{p_o - p_i}{t}$ which requires $n$ is large enough, $\Pi(n, p_o, p_i)$ is decreasing in $n$, so $\frac{1}{2n} < \frac{p_o - p_i}{t}$ is dominated by $\frac{p_o - p_i}{t} \leq \frac{1}{2n} < \frac{v - p_i}{t}$. If $\frac{p_o - p_i}{t} \leq \frac{1}{2n} < \frac{v - p_i}{t}$, since $\frac{d}{dn} = -2\alpha \mu (p_o - p_i)^2 n - s < 0$, $\frac{p_o - p_i}{t} \leq \frac{1}{2n} < \frac{v - p_i}{t}$ is dominated by $\frac{1}{2n} \geq \frac{p_o - p_i}{t}$. Thus, the optimal strategy can only be supported by $\frac{1}{2n} \geq \frac{v - p_i}{t}$ in Case 2.1.
In Case 2.2, depending on $n$, the profit function becomes

$$\Pi_{2.2}(n, p_o, p_i) = \begin{cases} 
(p_i - c)\mu - sn - k & \text{if } \frac{1}{2n} < \frac{v - p_i}{t}, \\
(p_o - c)\alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right) + (p_i - c) \left[ \alpha \mu \left( \frac{p_o - p_i}{t} \right) + (1 - \alpha) \mu \right] - sn - k & \text{if } \frac{1}{2n} \leq \frac{1}{2n} < \frac{p_o - p_i + e}{t}, \\
(p_o - c) \left[ \alpha \mu \left( \frac{1}{2n} - \frac{p_o - p_i}{t} \right) + (1 - \alpha - \beta) \mu \left( \frac{v - p_i}{t} \right) \right] - sn - k & \text{if } \frac{1}{2n} \geq \frac{v - p_i}{t}.
\end{cases}$$

It can be easily shown that $\Pi(n, p_o, p_i)$ is decreasing in $n$ if $\frac{1}{2n} < \frac{v - p_i}{t}$. Thus, the optimal strategy can only be supported by $\frac{1}{2n} \geq \frac{v - p_i}{t}$ in Case 2.2.

In Case 2.3, depending on $n$, the profit function becomes

$$\Pi_{2.3}(n, p_o, p_i) = \begin{cases} 
(p_o - c)\alpha \mu + (p_i - c)(1 - \alpha) \mu - sn - k & \text{if } \frac{1}{2n} < \frac{v - p_i}{t}, \\
(p_o - c)\alpha \mu + (p_i - c)(1 - \alpha) \mu \left( \frac{v - p_i}{t} \right) - sn - k & \text{if } \frac{1}{2n} \geq \frac{v - p_i}{t}.
\end{cases}$$

Since $\Pi(n, p_o, p_i)$ is decreasing in $n$ if $\frac{1}{2n} < \frac{v - p_i}{t}$, the optimal strategy can only be supported by $\frac{1}{2n} \geq \frac{v - p_i}{t}$ in Case 2.3.

In Case 2.4, depending on $n$, the profit function becomes

$$\Pi_{2.4}(n, p_o, p_i) = \begin{cases} 
(p_o - c)\alpha \mu + (p_i - c)(1 - \alpha) \mu - sn - k & \text{if } \frac{1}{2n} < \frac{p_o - p_i + e}{t}, \\
(p_o - c) \left[ \alpha \mu + (1 - \alpha) \mu \left( \frac{p_o - p_i + e}{t} \right) \right] - sn - k & \text{if } \frac{1}{2n} \leq \frac{1}{2n} < \frac{v - p_i}{t}, \\
(p_o - c) \left[ \alpha \mu + (1 - \alpha - \beta) \mu \left( \frac{v - p_i}{t} \right) \right] - sn - k & \text{if } \frac{1}{2n} \geq \frac{v - p_i}{t}.
\end{cases}$$

Since $\Pi(n, p_o, p_i)$ is decreasing in $n$ if $\frac{1}{2n} < \frac{v - p_i}{t}$, the optimal strategy can only be supported by $\frac{1}{2n} \geq \frac{v - p_i}{t}$ in Case 2.4.
In Case 2.5, depending on \( n \), the profit function becomes
\[
\Pi_{2.5}(n,p_o,p_i) = \begin{cases} 
(p_o - c)(\alpha + \beta)\mu + (p_i - c)(1 - \alpha - \beta)\mu - sn - k & \text{if } \frac{1}{2n} < \frac{v - p_i}{t}, \\
(p_o - c)(\alpha + \beta)\mu + (p_i - c)(1 - \alpha - \beta)\mu \left(\frac{v - p_i}{t}\right)2n - sn - k & \text{if } \frac{1}{2n} \geq \frac{v - p_i}{t}.
\end{cases}
\]
Since \( \Pi(n,p_o,p_i) \) is decreasing in \( n \) if \( \frac{1}{2n} < \frac{v - p_i}{t} \), the optimal strategy can only be supported by \( \frac{1}{2n} > \frac{v - p_i}{t} \) in Case 2.5.

Now we optimize \( n \). Note that \( \Pi(n,p_o,p_i) \) is piecewise linear in \( n \) in all subcases. In Case 2.1, \( n^1(p_o,p_i) = \frac{t}{2(v-p_i)} \), and the profit function reduces to
\[
\Pi_{2.1}(p_o,p_i) = (p_o - c)\alpha\mu \left(\frac{v - p_o}{v - p_i}\right) + (p_i - c) \left[ \alpha\mu \left(\frac{p_o - p_i}{v - p_i}\right) + (1 - \alpha)\mu \right] - \frac{st}{2(v-p_i)} - k.
\] (B.11)

In Case 2.2, \( n^1(p_o,p_i) = \frac{t}{2(v-p_i)} \), and the profit function reduces to
\[
\Pi_{2.2}(p_o,p_i) = (p_o - c) \left[ \alpha\mu \left(\frac{v - p_o}{v - p_i}\right) + \beta\mu \left(\frac{v - p_o - e}{v - p_i}\right) \right] + (p_i - c) \left[ \alpha\mu \left(\frac{p_o - p_i}{v - p_i}\right) + \beta\mu \left(\frac{p_o - p_i + e}{v - p_i}\right) + (1 - \alpha - \beta)\mu \right] - \frac{st}{2(v-p_i)} - k.
\] (B.12)

In Case 2.3, \( n^1(p_o,p_i) = \frac{t}{2(v-p_i)} \), and the profit function reduces to
\[
\Pi_{2.3}(p_o,p_i) = (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu - \frac{st}{2(v-p_i)} - k.
\]

Since \( \Pi_{2.3}(p_o,p_i) \) is increasing in \( p_o \), Case 2.3 is dominated by Case 2.1. In Case 2.4, the optimal \( n \) is either \( \frac{t}{2(v-p_i)} \) or \( \frac{t}{2(v-p_i) + v} \). With \( n = \frac{t}{2(v-p_i)} \), the profit function reduces to
\[
\Pi_{2.4}(p_o,p_i) = (p_o - c) \left[ \alpha\mu + \beta\mu \left(\frac{v - p_o - e}{v - p_i}\right) \right] + (p_i - c) \left[ \beta\mu \left(\frac{p_o - p_i + e}{v - p_i}\right) + (1 - \alpha - \beta)\mu \right] - \frac{st}{2(v-p_i)} - k.
\] (B.13)

With \( n = \frac{t}{2(v-p_i) + v} \), the profit function reduces to
\[
\Pi_{2.4}(p_o,p_i) = (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu - \frac{st}{2(p_o - p_i + e)} - k
\]
which is increasing in \( p_o \), so with \( n = \frac{t}{2(v-p_i) + v} \), Case 2.4 is dominated by either Case 2.2 or Case 2.3. Thus, the optimal strategy can only be supported by \( n^1(p_o,p_i) = \frac{t}{2(v-p_i)} \) in Case 2.4. Finally, in Case 2.5, \( n^1(p_o,p_i) = \frac{t}{2(v-p_i)} \), and the profit function reduces to
\[
\Pi_{2.5}(p_o,p_i) = (p_o - c)(\alpha + \beta)\mu + (p_i - c)(1 - \alpha - \beta)\mu - \frac{st}{2(v-p_i)} - k.
\]

Since \( \Pi_{2.5}(p_o,p_i) \) is increasing in \( p_o \), Case 2.5 is dominated by either Case 2.3 or Case 2.4.

So far, we have seen that Case 2.3 and Case 2.5 are not optimal. Next, we optimize \( p_o \) by considering \( \Pi_{2.1}(p_o,p_i), \Pi_{2.2}(p_o,p_i), \) and \( \Pi_{2.4}(p_o,p_i) \). First, consider Case 2.4. Taking derivative of (B.13) with respect to \( p_o \) yields
\[
\frac{\partial\Pi_{2.4}}{\partial p_o} = \alpha\mu + \beta\mu \left(\frac{v - 2p_o + p_i - e}{v - p_i}\right)
\]
which is decreasing in \( p_o \), so \( \Pi_{2.4}(p_o,p_i) \) is concave in \( p_o \). For \( p_i < v - e \), Case 2.4 intersects with Case 2.2 at \( p_o = p_i \). Then, since
\[
\frac{\partial\Pi_{2.4}}{\partial p_o} \bigg|_{p_o=p_i} = \alpha\mu + \beta\mu \left(\frac{v - p_i - e}{v - p_i}\right) > 0,
\]
Case 2.4 is dominated by Case 2.2. For \( p_i \geq v - e \), Case 2.4 intersects with Case 2.3 at \( p_o = v - e \). Then, since
\[
\frac{\partial \Pi_{2.4}}{\partial p_o} \bigg|_{p_o=v-e} = \alpha \mu + \beta \mu \left( \frac{-v + e + p_i}{v - p_i} \right) > 0,
\]
Case 2.4 is dominated by Case 2.3. Thus, Case 2.4 is not optimal.

Second, consider Case 2.2. Taking derivative of (B.12) with respect to \( p_o \) yields
\[
\frac{\partial \Pi_{2.2}}{\partial p_o} = \alpha \mu \left( \frac{v - 2p_o + p_i}{v - p_i} \right) + \beta \mu \left( \frac{v - 2p_o + p_i - e}{v - p_i} \right)
\]
which is decreasing in \( p_o \), so \( \Pi_{2.2}(p_o, p_i) \) is concave in \( p_o \). Solving the first-order condition yields \( p_o = \frac{v + p_i}{2} - \frac{\beta e}{2(\alpha + \beta)} \). Recall that Case 2.2 requires \( p_i < p_o < v - e \). Also, note that Case 2.2 is valid only for \( p_i < v - e \). Then, we have \( \frac{v + p_i}{2} - \frac{\beta e}{2(\alpha + \beta)} > \frac{v + p_i}{2} - \frac{v - e}{2} = \frac{v - e + p_i}{2} + p_i > p_i \). Moreover, \( \frac{v + p_i}{2} - \frac{\beta e}{2(\alpha + \beta)} < v - e \) is equivalent to \( p_i < v - e - \frac{\alpha e}{\alpha + \beta} \). Thus, for \( p_i \geq v - e - \frac{\alpha e}{\alpha + \beta} \), \( p_o^i(p_i) = v - e \). For \( p_i < v - e - \frac{\alpha e}{\alpha + \beta} \), \( p_o^i(p_i) = \frac{v + p_i}{2} - \frac{\beta e}{2(\alpha + \beta)} \), and the profit function reduces to
\[
\Pi_{2.2}(p_i) = (p_i - c)\mu + \frac{(\alpha + \beta)\mu(v - p_i)}{4} - \frac{\beta \mu e}{2} + \frac{(\beta \mu e)^2}{4(\alpha + \beta)} - \frac{st}{2} \cdot \frac{1}{v - p_i} - k. \tag{B.14}
\]

Third, consider Case 2.1. Taking derivative of (B.11) with respect to \( p_o \) yields
\[
\frac{\partial \Pi_{2.1}}{\partial p_o} = \alpha \mu \left( \frac{v - 2p_o + p_i}{v - p_i} \right)
\]
which is decreasing in \( p_o \), so \( \Pi_{2.1}(p_o, p_i) \) is concave in \( p_o \). Solving the first-order condition yields \( p_o = \frac{v + p_i}{2} \). Recall that Case 2.1 requires \( p_o \geq v - e, \frac{v + p_i}{2} > v - e \) is equivalent to \( p_i > v - 2e \). Thus, for \( p_i \leq v - 2e \), \( p_o^i(p_i) = v - e \). For \( p_i > v - 2e \), \( p_o^i(p_i) = \frac{v + p_i}{2} \), and the profit function reduces to
\[
\Pi_{2.1}(p_i) = (p_i - c)\mu + \frac{\alpha \mu(v - p_i)}{4} - \frac{st}{2(v - p_i)} - k. \tag{B.15}
\]

In optimizing \( p_o \), we know that the optimal strategy can only be achieved in Case 2.1 or Case 2.2. Moreover, for \( p_i \leq v - 2e \), Case 2.1 is dominated by Case 2.2. For \( p_i \geq v - e - \frac{\alpha e}{\alpha + \beta} \), Case 2.2 is dominated by Case 2.1. Note that \( v - 2e < v - e - \frac{\alpha e}{\alpha + \beta} \).

Next, we optimize \( p_i \) and characterize the optimal strategy. First, consider Case 2.1. Taking derivative of (B.15) with respect to \( p_i \) yields
\[
\frac{d\Pi_{2.1}}{dp_i} = \left( 1 - \frac{\alpha}{4} \right) \mu - \frac{st}{2(v - p_i)^2}
\]
which is decreasing in \( p_i \), so \( \Pi_{2.1}(p_i) \) is concave in \( p_i \). If
\[
\frac{d\Pi_{2.1}}{dp_i} \bigg|_{p_i=v-2e} = \left( 1 - \frac{\alpha}{4} \right) \mu - \frac{st}{8e^2} > 0,
\]
or equivalently,
\[
e > \sqrt{\frac{st}{8(1 - \frac{\alpha}{4})}} \mu \overset{\text{def}}{=} e_1,
\]
the optimal \( p_i \) in Case 2.1 is given by the first-order condition. Solving the first-order condition yields
\[
p_o^i = v - \sqrt{\frac{st}{2(1 - \frac{\alpha}{4})}} \mu.
\]
Correspondingly,
\[
\Pi_{2.1}^i = (v - c)\mu - \sqrt{2\left(1 - \frac{\alpha}{4}\right)st\mu} - k. \tag{B.16}
\]
On the other hand, if \( e \leq e_1 \), Case 2.1 is dominated by Case 2.2.

Second, consider Case 2.2. Taking derivative of (B.14) with respect to \( p_i \) yields

\[
\frac{d\Pi_{22}}{dp_i} = \left( 1 - \frac{\alpha + \beta}{4} \right) \mu + \left[ \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} - \frac{st}{2} \right] \frac{1}{(v - p_i)^2}.
\]

If \( \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} - \frac{st}{2} > 0 \), then Case 2.2 is dominated by Case 2.1. If \( \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} - \frac{st}{2} \leq 0 \), \( \Pi_{22}(p_i) \) is concave in \( p_i \).

Thus, if \( \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} - \frac{st}{2} > 0 \), Case 2.2 is dominated by Case 2.1. If \( \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} - \frac{st}{2} \leq 0 \), we have shown that if \( \Pi_{21} \geq \Pi_{22} \) is equivalent to

\[
\alpha \mu e \left[ \frac{2st - \beta^2 \mu e^2}{\alpha + \beta} \right] \mu - \frac{\beta \mu e}{2} - k. \tag{B.17}
\]

\( \Pi_{22} = (v - e)\mu - \sqrt{\left( 1 - \frac{\alpha + \beta}{4} \right) \left( 2st - \frac{\beta^2 \mu e^2}{\alpha + \beta} \right) \mu - \frac{\beta \mu e}{2}}. \tag{B.18}
\]

Note that (B.17) is equivalent to

\[
e < \left( \alpha + \beta \right) \sqrt{\frac{2st}{(4 - \alpha - \beta)(2\alpha + \beta)^2 + \beta^2(\alpha + \beta)}} \mu. \]

Thus, if \( e < e_2 \), \( p_i^1 \) and \( \Pi_{22}^1 \) are given above. On the other hand, if \( e \geq e_2 \), Case 2.2 is dominated by Case 2.1.

We have shown that if \( e_1 < e < e_2 \), the optimal \( p_i \) is given by the first-order condition in both Case 2.1 and Case 2.2, and the profits are given by (B.16) and (B.18), respectively. From (B.16) and (B.18), we know \( \Pi_{21} \geq \Pi_{22} \) is equivalent to

\[
\sqrt{2 \left( 1 - \frac{\alpha}{4} \right) st \mu - \frac{\beta \mu e}{2}} \leq \sqrt{\left( 1 - \frac{\alpha + \beta}{4} \right) \left( 2st - \frac{\beta^2 \mu e^2}{\alpha + \beta} \right) \mu}. \tag{B.19}
\]

Note that \( e < e_2 \) implies \( \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} - \frac{st}{2} \leq 0 \) which in turn implies \( \sqrt{2 \left( 1 - \frac{\alpha}{4} \right) st \mu - \frac{\beta \mu e}{2}} > 0 \). Taking square on both sides of (B.19) and simplifying the resulting inequality yields

\[
\frac{\beta \mu}{\alpha + \beta} \cdot e^2 - \sqrt{2 \left( 1 - \frac{\alpha}{4} \right) st \mu \cdot e + \frac{st}{2}} \leq 0. \tag{B.20}
\]

Solving (B.20) yields \( e_3 \leq e \leq e_4 \), where

\[
e_3 = \frac{\sqrt{st}}{\sqrt{2 \left( 1 - \frac{\alpha}{4} \right) \mu + \sqrt{2 \left( 1 - \frac{\alpha}{4} \right) \mu - \frac{3 \mu}{\alpha + \beta}}}}.
\]

\[
e_4 = \frac{\sqrt{st}}{\sqrt{2 \left( 1 - \frac{\alpha}{4} \right) \mu + \sqrt{2 \left( 1 - \frac{\alpha}{4} \right) \mu - \frac{3 \mu}{\alpha + \beta}}}}
\]

To determine when Case 2.1 or Case 2.2 is optimal, we show that the following three conditions hold: 1) \( e_3 < e_2 \), 2) \( e_4 > e_1 \), 3) \( e_4 > e_2 \). Given these conditions, Case 2.1 is optimal if and only if \( e \geq e_3 \), and Case 2.2 is optimal if and only if \( e < e_3 \). First, \( e_3 < e_2 \) is equivalent to

\[
\sqrt{(4 - \alpha - \beta)(2\alpha + \beta)^2 + \beta^2(\alpha + \beta)} < \sqrt{\alpha + \beta} \left[ \sqrt{(\alpha + \beta)(4 - \alpha)} + \sqrt{(4 - \alpha - \beta)} \right]. \tag{B.21}
\]
Taking square on both sides of (B.21) and rearranging terms yields
\[ \alpha \left[ 1 + 2\alpha - (1 - \alpha - \beta)^2 \right] < (\alpha + \beta)\sqrt{(\alpha + \beta)(4 - \alpha)\alpha(4 - \alpha - \beta)}. \] (B.22)

Then, taking square on both sides of (B.22) and rearranging terms yields
\[ 4\alpha\beta \left[ -2\alpha^2(1 - \alpha) - (5\alpha^2 + 4\alpha\beta + \beta^2)(1 - \beta) - \alpha^2 - 7\alpha\beta - 3\beta^2 \right] < 0 \]
which is true. Second, \( e_3 > e_1 \) holds because
\[ e_3 > \frac{\sqrt{st}}{2 \sqrt{2 (1 - \frac{\alpha}{4}) s t \mu}} = e_1. \]

Third, to show \( e_4 > e_2 \), it suffices to show
\[ \frac{\alpha + \beta}{2\beta \mu} \sqrt{2 \left( 1 - \frac{\alpha}{4} \right) s t \mu} > (\alpha + \beta) \sqrt{\frac{2st}{(4 - \alpha - \beta)(2\alpha + \beta)^2 + \beta^2(\alpha + \beta)}} \mu, \]
which can be simplified to
\[ \sqrt{4 - \alpha} \cdot \sqrt{(4 - \alpha - \beta)(2\alpha + \beta)^2 + \beta^2(\alpha + \beta)} > 4\beta. \] (B.23)

Taking square on both sides of (B.23) and rearranging terms yields
\[ 4\alpha \left[ \alpha(\alpha + \beta)^2 + 7\alpha^2 + (8\alpha + 11\beta)(1 - \alpha) + (\alpha + 5\beta)(1 - \beta) \right] > 0 \]
which is true.

Therefore, we have shown that Case 2.1 is optimal if \( e \geq \bar{e} \) and Case 2.2 is optimal if \( e < \bar{e} \). When Case 2.1 is optimal, tracing back our analysis for Case 2.1, we obtain that \( p_u^1 = \frac{\nu + p_l^1}{2} \) and \( n^1 = \frac{\tau}{2(\nu - p_l^1)} \), which are the ones shown in Proposition B.2(i). It is easy to see that \( p_u^1 > p_l^1 \). Under the optimal strategy, Type I customers purchase online if \( \frac{p_{l1} - p_{l1}^1}{t} \leq x \leq \frac{1}{2s} \) and purchase in-store if \( 0 \leq x < \frac{p_{l1} - p_{l1}^1}{t} \), all Type II and Type III customers purchase in-store.

Moreover, when Case 2.2 is optimal, tracing back our analysis for Case 2.2, we obtain that \( p_u^1 = \frac{\nu + p_l^1}{2} - \frac{se}{2(\alpha + \beta)} \) and \( n^1 = \frac{\tau}{2(\nu - p_l^1)} \), which are the ones shown in Proposition B.2(ii). \( p_u^1 > p_l^1 \) can be simplified to
\[ e < \frac{\alpha + \beta}{\beta} \sqrt{\frac{st}{2 \mu}}. \] (B.24)

To show (B.24) is true if \( e < \bar{e} \), it suffices to show
\[ \bar{e} < \frac{\alpha + \beta}{\beta} \sqrt{\frac{st}{2 \mu}}. \]

Then, since
\[ \bar{e} < \sqrt{\frac{st}{2 (1 - \frac{1}{4}) \mu}}, \]
it suffices to show
\[ \sqrt{\frac{st}{2 (1 - \frac{1}{4}) \mu}} < \frac{\alpha + \beta}{\beta} \sqrt{\frac{st}{2 \mu}}. \] (B.25)

Taking square on both sides of (B.25) and simplifying the resulting inequality yields \( \alpha \left[ - (\alpha + \beta)^2 + 4\alpha + 8\beta \right] > 0 \) which is true. Thus, \( p_u^1 > p_l^1 \). Finally, under the optimal strategy, Type I customers purchase online if \( \frac{p_{l1} - p_{l1}^1}{t} \leq x \leq \frac{1}{2s} \) and purchase in-store if \( 0 \leq x < \frac{p_{l1} - p_{l1}^1}{t} \), Type II customers purchase online if \( \frac{p_{l1} - p_{l1}^1 + \epsilon}{t} \leq x \leq \frac{1}{2s} \) and purchase in-store if \( 0 \leq x < \frac{p_{l1} - p_{l1}^1 + \epsilon}{t} \), and all Type III customers purchase in-store. The proof is complete. \( \square \)
Proof of Proposition B.3  Using the results in Propositions B.1 and B.2, we prove Proposition B.3 as follows:

(i) It is easy to see $n^l < n^*$ if $e \geq \bar{e}$. If $e < \bar{e}$, $n^l < n^*$ can be simplified to

$$e < \frac{\alpha + \beta}{\beta} \sqrt{\frac{st}{2\mu}}$$

which is same as (B.24). Thus, $n^l < n^*$.

(ii) It is easy to see $p^l_o > p^*_o$ if $e \geq \bar{e}$. If $e < \bar{e}$, $p^l_o > p^*_o$ is equivalent to

$$(\alpha + \beta)\sqrt{\frac{st\mu}{2}} \geq \frac{(\alpha + \beta)\mu}{2} \sqrt{\frac{\frac{\alpha + \beta - \beta e}{4(\alpha + \beta)}}{\mu} \frac{(\alpha + \beta - \beta e)}{4(\alpha + \beta)}} > \frac{\beta e}{2}.$$  (B.27)

From (B.19), we know that $e < \bar{e}(= e_3)$ implies

$$\sqrt{2 \left(1 - \frac{\alpha}{4}\right) st\mu} > \sqrt{\left(1 - \frac{\alpha + \beta}{4}\right) \left(2 st - \frac{\beta^2 \mu e^2}{2(\alpha + \beta)}\right) > \frac{\beta e}{2}.$$  (B.28)

For (B.28) to sufficiently imply (B.27), we need the left-hand side of (B.27) to be larger than the left-hand side of (B.28), which can be simplified to the following:

$$(\sqrt{4 - \alpha - \alpha - \beta}) \sqrt{\left(1 - \frac{\alpha + \beta}{4}\right) st\mu} < (2 - \alpha - \beta) \sqrt{\frac{st\mu}{2} - \frac{\beta^2 \mu e^2}{4(\alpha + \beta)}}.$$  (B.29)

First, it is easy to see $\sqrt{4 - \alpha - \alpha - \beta} < 2 - \alpha - \beta$. Second, (B.26) implies

$$\sqrt{\left(1 - \frac{\alpha + \beta}{4}\right) st\mu} < \sqrt{\frac{st\mu}{2} - \frac{\beta^2 \mu e^2}{4(\alpha + \beta)}}.$$  

Thus, $p^l_o > p^*_o$.

(iii) It is easy to see $p^l_i < p^*_i$ if $e \geq \bar{e}$. If $e < \bar{e}$, $p^l_i < p^*_i$ can be simplified to (B.26) which we have shown is true. Thus, $p^l_i < p^*_i$.

(iv) $z^l < z^*$ follows from $p^l_o < p^*_o$.

(v) Since the firm sells to all customers under the optimal strategy in both cases, it suffices to consider the online demand. It is easy to see that if $e \geq \bar{e}$, the firm’s online demand is lower in Case 2 than in Case 1. If $e < \bar{e}$, the condition for the online demand to be lower in Case 2 than in Case 1 is

$$\alpha > \frac{\alpha + \beta}{2} - \beta e \sqrt{\frac{1 - \frac{\alpha + \beta}{4}}{2st - \frac{\beta^2 \mu e^2}{4(\alpha + \beta)}}}$$

which is equivalent to

$$(\beta - \alpha) \sqrt{\frac{st\mu}{2}} - \frac{\beta^2 \mu e^2}{4(\alpha + \beta)} < \beta e \sqrt{\frac{1 - \frac{\alpha + \beta}{4}}{2st - \frac{\beta^2 \mu e^2}{4(\alpha + \beta)}}}.$$  (B.30)

If $\beta < \alpha$, (B.30) holds trivially. If $\beta \geq \alpha$, taking square on both sides of (B.30) and simplifying the resulting inequality yields

$$e > \frac{\beta - \alpha}{\beta} \sqrt{\frac{(\alpha + \beta)st}{2(\alpha + \beta - \alpha \beta)\mu}}.$$  (B.31)

Note that if $\beta < \alpha$, the right-hand side of (B.31) is negative and hence (B.30) holds. Thus, (B.30) holds if and only if $e > \bar{e}$. Finally, since (B.30) holds when $e = \bar{e}$, we know $e < \bar{e}$.
(vi) By comparing \( \Pi^* \) and \( \Pi^1 \), it is easy to see that \( \Pi^1 \geq \Pi^* \) if and only if \( k \leq \bar{k} \). Moreover, if \( e \geq \bar{e} \), it is easy to see that \( \bar{k} > 0 \). If \( e < \bar{e} \), from (B.19) we know that
\[
\sqrt{2 \left( 1 - \frac{\alpha}{4} \right)} s \mu - \sqrt{\left( 1 - \frac{\alpha + \beta}{4} \right) \left( 2st - \frac{\beta^2 \mu e^3}{\alpha + \beta} \right) \mu - \frac{\beta \mu e}{2}} > 0,
\]
which implies \( \bar{k} > 0 \).

(vii) If \( e \geq \bar{e} \),
\[
\Pi^1 - \Pi^* = \sqrt{2st\mu} \left( 1 - \sqrt{1 - \frac{\alpha}{4}} \right) - k
\]
which is increasing in \( t \) and \( s \). If \( e < \bar{e} \),
\[
\Pi^1 - \Pi^* = \sqrt{2st\mu} - \sqrt{\left( 1 - \frac{\alpha + \beta}{4} \right) \left( 2st - \frac{\beta^2 \mu e^3}{\alpha + \beta} \right) \mu - \frac{\beta \mu e}{2} - e}.
\]
Then,
\[
\frac{\partial(\Pi^1 - \Pi^*)}{\partial t} = \frac{s}{t} \left[ \sqrt{\frac{\mu t}{2s}} - \sqrt{\frac{1 - \frac{\alpha + \beta}{4} \mu t}{2s - \frac{\beta^2 \mu e^3}{\alpha + \beta} \mu}} \right] = \frac{s}{t} (n^* - n^1) > 0.
\]
Moreover, since \( \Pi^1 - \Pi^* \) is symmetric in \( t \) and \( s \), \( \frac{\partial(n^* - n^1)}{\partial s} > 0. \)

Proof of Proposition B.4. To analyze the profit function and derive the optimal strategy, we define three subcases in Case 3: Case 3.1 \( (p_i - p_o \geq e) \), Case 3.2 \( (0 \leq p_i - p_o < e) \), and Case 3.3 \( (p_i - p_o < 0) \). When \( \sigma = 0 \), based on the demand segmentation in §4.3, the profit functions in each subcase are
\[
\Pi_{3.1}(p_o, p_i) = (p_o - c)(\alpha + \beta)\mu + (p_i - c)(1 - \alpha - \beta)\mu - k - k',
\]
\[
\Pi_{3.2}(p_o, p_i) = (p_o - c)\alpha\mu + (p_i - c)(1 - \alpha)\mu - k - k',
\]
\[
\Pi_{3.3}(p_o, p_i) = (p_i - c)\mu - k - k',
\]
respectively.

In Case 3.1 which requires \( p_o \) is small enough, \( \Pi_{3.1}(p_o, p_i) \) is increasing in \( p_o \), so the optimal strategy in Case 3.1 is achieved by \( p_i - p_o = e \). When \( p_i - p_o = e \), we have
\[
\Pi_{3.1}(p_o, p_i) = (p_o - c)(1 - \alpha - \beta)\mu e, \\
\Pi_{3.2}(p_o, p_i) = (p_o - c)(1 - \alpha)\mu e,
\]
so \( \Pi_{3.1}(p_o, p_i) < \Pi_{3.2}(p_o, p_i) \). Thus, Case 3.1 is dominated by Case 3.2. In Case 3.3 which requires \( p_i \) is small enough, \( \Pi_{3.3}(p_o, p_i) \) is increasing in \( p_i \), so Case 3.3 is also dominated by Case 3.2. Thus, the optimal strategy is achieved in Case 3.2. In Case 3.2, \( \Pi_{1.2}(p_o, p_i) \) is increasing in both \( p_o \) and \( p_i \). Thus, the optimal prices are \( p_o^* = p_i^* = v \). Correspondingly, the optimal profit is \( \Pi^1 = (v - c)\mu - k - k' \). Then, the demand segmentation under the optimal strategy follows directly. □

Proof of Proposition B.5 The proof is similar to the proof of Proposition B.3, and is hence omitted. □
Appendix C: Additional Contracting Analysis for 3D Printing In-Store and Proofs

In this section, we consider a (decentralized) supply chain with 3D printing in-store where the retailer controls the raw material inventory. Consistent with §4.4, for ease of notation, we shall drop the subscript “i” from all corresponding functions and variables.

In this case, the retailer chooses the inventory order quantity $q$, and the two parties negotiate on the wholesale price $w$. The raw material cost is incurred by the supplier, while the production cost is incurred by the retailer. The retailer’s profit is

$$\Pi_R(q, w) = (v - c_p)E \left[ \min(D, q) \right] - wq,$$

and the supplier’s profit is

$$\Pi_S(q, w) = (w - c_r)q.$$

Under the wholesale contract, for any wholesale price $w$, the retailer’s optimal inventory order quantity is $q^*_R = d^\mu + z^*_R \sigma \sqrt{d^\mu}$ where

$$z^*_R = \Phi^{-1} \left( 1 - \frac{w}{v - c_p} \right).$$

The subscript “$R$” indicates retailer managing the inventory and the superscript “$w$” indicates wholesale contract. Thus, by comparing (C.1) to (8), we can see that as long as $w > c_r$, there exists loss of efficiency in the decentralized supply chain. Although the raw material inventory would be less than that under the centralized system no matter who makes the inventory decision, the reasons are different. When the retailer controls inventory, it is because the retailer’s raw material procurement cost (i.e., the wholesale price) is greater than that under the centralized system. When the supplier controls inventory, it is because the supplier’s share of retail profit (which now depends on the wholesale price) is less than that under the integrated system.

With 3D printing in-store, if the retailer controls the raw material inventory, the setting is similar to the supply chain contracting setting in existing literature. The difference here is that the retailer’s production cost is incurred after demand realization. We show in Proposition C.1 that contracts such as buy-back and revenue-sharing can still coordinate the supply chain. Let $b$ be the buy-back price under the buy-back contract, and $\gamma$ be the retailer’s share of revenue generated from each unit under the revenue-sharing contract. The parties negotiate on $(w, b)$ under a buy-back contract and $(w, \gamma)$ under a revenue-sharing contract. Then, given the contracting parameters, the retailer makes inventory decisions.

Under the buy-back contract, the parties’ profit functions are (the superscript “$b$” indicates buy-back contract)

$$\Pi^b_R(q, w, b) = (v - c_p - b)E \left[ \min(D, q) \right] - (w - b)q,$$

$$\Pi^b_S(q, w, b) = bE \left[ \min(D, q) \right] - (c_r - w + b)q.$$

Under the revenue-sharing contract, the parties’ profit functions are (the superscript “$r$” indicates revenue-sharing contract)

$$\Pi^r_R(q, w, \gamma) = (\gamma v - c_p)E \left[ \min(D, q) \right] - wq,$$

$$\Pi^r_S(q, w, \gamma) = (1 - \gamma)vE \left[ \min(D, q) \right] - (c_r - w)q.$$
Proposition C.1. (i) If the retailer controls the raw material inventory, the supply chain can be coordinated under the set of buy-back contracts \((w, b)\) with

\[
b = w \cdot \frac{v - c_p}{v - c} - \frac{(v - c_p) c_r}{v - c}.
\]  
(C.2)

Under these contracts, the retailer’s optimal inventory order quantity is equal to the optimal quantity in the centralized case, and the retailer’s optimal profit is \(\frac{v - c_p - b}{v - c_p} \cdot \Pi(q^i)\).

(ii) The supply chain can also be coordinated under the set of revenue-sharing contracts \((w, \gamma)\) with

\[
\gamma = w \cdot \frac{v - c_p}{v c_r} + \frac{c_r}{v}.
\]  
(C.3)

Under these contracts, the retailer’s optimal inventory order quantity is equal to the optimal quantity in the centralized case, and the retailer’s optimal profit is \(\frac{\gamma v - c_p}{v c_p} \cdot \Pi(q^i)\).

(iii) For any revenue-sharing contract \((w_0, \gamma)\) that coordinates the supply chain, there exists a unique buy-back contract \((w, b) = (w_0 + (1 - \gamma) v, (1 - \gamma) v)\) that generates the same profit for each party for any realization of demand.

(iv) For any cost-sharing contract \((w_0, \beta)\) that coordinates the supply chain where the supplier controls the raw material inventory, there exist a unique buy-back contract \((w, b) = (w_0 + \beta c_r, w_0)\) and a unique revenue-sharing contract \((w, \gamma) = (\beta c_r, 1 - \frac{c_r}{v})\) that generate the same profit for each firm for any realization of demand in the scenario where the retailer controls the raw material inventory.

Proof of Proposition C.1. (i) Under the buy-back contract, for any \((w, b)\), the retailer’s optimal inventory order quantity is \(q^b_R = d^i \mu + z^b_R \sigma \sqrt{d^i}\) where

\[
q^b_R = \Phi^{-1} \left( 1 - \frac{w - b}{v - c_p - b} \right).
\]  
(C.4)

The decentralized supply chain can be coordinated if \(z^b_R = z^i\). From (8) and (C.4), we obtain that \(z^b_R = z^i\) requires \(\frac{w - b}{v - c_p - b} = \frac{c_r}{v - c_p}\) which is equivalent to (C.2). Then, replacing \(w - b\) by \(\frac{v - c_p - b}{v - c_p} \cdot c_r\), we have

\[
\Pi^b_R(q^b_R, w, \gamma) = \frac{v - c_p - b}{v - c_p} \cdot \Pi(q^i).
\]

(ii) Under the revenue-sharing contract, for any \((w, \gamma)\), the retailer’s optimal inventory order quantity is \(q^\gamma_R = d^i \mu + z^\gamma_R \sigma \sqrt{d^i}\) where

\[
z^\gamma_R = \Phi^{-1} \left( 1 - \frac{w}{\gamma v - c_p} \right).
\]  
(C.5)

The decentralized supply chain can be coordinated if \(z^\gamma_R = z^i\). From (8) and (C.5), we obtain that \(z^\gamma_R = z^i\) requires \(\frac{w}{\gamma v - c_p} = \frac{c_r}{v - c_p}\) which is equivalent to (C.3). Then, replacing \(w\) by \(\frac{\gamma v - c_p}{v - c_p} \cdot c_r\), we have

\[
\Pi^\gamma_R(q^\gamma_R, w, \gamma) = \frac{\gamma v - c_p}{v - c_p} \cdot \Pi(q^i).
\]

(iii) Under the revenue-sharing contract, the retailer’s profit share is \(\frac{\gamma v - c_p}{v - c_p}\), and under the buy-back contract, the retailer’s profit share is \(\frac{v - c_p - b}{v - c_p}\). By making \(\frac{\gamma v - c_p}{v - c_p} = \frac{v - c_p - b}{v - c_p}\), we obtain \(b = (1 - \gamma) v\). Moreover, a coordinating buy-back contract requires (C.2). Replacing \(b\) by \((1 - \gamma) v\) in (C.2) and using (C.3) to simplify, we obtain \(w = w_0 + b = w_0 + (1 - \gamma) v\).

(iv) We showed in the proof of Proposition 6 that under the cost-sharing contract, the supplier’s profit share is \(1 - \beta\). Proposition C.1(i) indicates that under the buy-back contract, the supplier’s profit share is \(1 - \frac{v - c_p - b}{v - c_p}\). By making \(1 - \frac{v - c_p - b}{v - c_p} = 1 - \beta\), we obtain \(b = (1 - \beta)(v - c_p)\). Moreover, we know from Proposition
C.1(i) that a coordinating buy-back contract requires (C.2). Replacing $b$ by $(1 - \beta)(v - c_p)$ in (C.2), we obtain $w = (1 - \beta)(v - c) + c_r$. Finally, since $w_0 = (1 - \beta)(v - c_p)$, we have $b = (1 - \beta)(v - c_p) = w_0$ and $w = (1 - \beta)(v - c) + c_r = (1 - \beta)(v - c_p) + \beta c_r = w_0 + \beta c_r$.

Similarly, Proposition C.1(ii) indicates that under the revenue-sharing contract, the supplier’s profit share is $1 - \frac{(v - c_p)}{v - c}$. By making $1 - \frac{(v - c_p)}{v - c} = 1 - \beta$, we obtain $\gamma = \beta + \frac{(1 - \beta)c_p}{v}$, Moreover, we know from Proposition C.1(ii) that a coordinating revenue-sharing contract requires (C.3). Replacing $\gamma$ by $\beta + \frac{(1 - \beta)c_p}{v}$ in (C.3), we obtain $w = \beta c_r$. Finally, since $w_0 = (1 - \beta)(v - c_p)$, we have $\gamma = \beta + \frac{(1 - \beta)c_p}{v} = 1 - \frac{(1 - \beta)(v - c_p)}{v} = 1 - \frac{w_0}{v}$. □

Proposition C.1 states that for both types of contract, there exist combinations of the two contracting parameters such that the retailer’s optimal inventory order quantity is same as the integrated system. Moreover, Proposition C.1(ii) states that under revenue-sharing contracts, the retailer’s share of supply chain profit is $\gamma v - c_p v - c_p$, which is smaller than (or equal to when $\gamma = 1$) its share of revenue. This is different from the result in Cachon and Lariviere (2005) that the share of profit is equal to the share of revenue, and is due to the fact that with 3D printing in-store, the production cost is incurred after demand realization just like the revenue, while the raw material cost is still incurred concurrently with inventory ordering. Proposition C.1(iii) replicates the equivalence result between buy-back and revenue-sharing contracts in Cachon and Lariviere (2005) in our 3D printing in-store case. For every coordinating revenue-sharing contract, there exists a buy-back contract that generates the same profit for each party.

For traditional supply chains, the use of revenue-sharing contracts is limited by the supplier’s ability to verify the retailer’s revenue. However, with 3D printing in-store, the retailer’s sales could be easily tracked by the 3D printer, so if the supplier owns the 3D printing technology, it could accurately track the retailer’s revenue. Therefore, it is quite reasonable to anticipate that revenue-sharing contracts can be more easily implemented in such a case. On the other hand, a coordinating revenue-sharing contract still requires the supplier to sell raw material below cost to the retailer, which may be difficult to implement in practice compared to buy-back contracts (Cachon and Lariviere 2005).

Proposition C.1(iv) further shows that for any coordinating cost-sharing contract when the supplier controls the raw material inventory, one can find a corresponding coordinating buy-back and revenue-sharing contract when the retailer controls the raw material inventory that achieves the same profit split in the supply chain. Therefore, regardless of who controls inventory, the supply chain can be coordinated in an equivalent way.
References


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