A Search and Learning Model of Export Dynamics

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2 sets of relevant issues

- Aggregate/industry level export dynamics
  - What determines short and long-run responses to macroeconomic shocks?
  - Why are export responses to trade liberalization unpredictable?
  - What are the underlying causes of export booms?

- Trade at the level of individual firms
  - What are the important firm-level trade frictions?
  - What determines the cross-firm distribution of export sales?
  - What determines firm-specific export growth patterns?
  - How reconcile the cross section and dynamic patterns?

This paper: Approach these issues by studying formation, evolution, and dissolution of international buyer-seller relationships.

Eaton et al. (\textsuperscript{a}Brown, \textsuperscript{b}NBER, \textsuperscript{c}U. de los Andes, \textsuperscript{d}U. Copenhagen, \textsuperscript{e}Census Bureau (CES), \textsuperscript{f}Penn State)
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The exercises

- Characterize buyer-seller relationships in 18 years worth of data on individual merchandise shipments from Colombia to the United States.
- Develop a (partial equilibrium) dynamic search and learning model that explains patterns found in shipments.
- Fit the model to the data, and quantify exporting frictions:
  - costs of finding new buyers
  - costs maintaining relationships with existing ones.
  - learning about product appeal in foreign markets
  - network effects
- Perform counterfactual exercises
Related literature

- Melitz (2003), etc.
  - More efficient firms more likely to export
  - More efficient firms sell more in any market

- Beachhead exporting costs:

- Marketing costs: Arkolakis (2009, 2010); Drozd and Nozal (2011)
- Learning: Rauch and Watson (2002); Albornoz, Calvo-Pardo, Corcos, and Ornelas (2012)
Patterns in the U.S. customs records

- Population of (legal) import shipments over the course of 18 years (1992-2009).
- Exclude affiliated party trade, non-manufactured goods.
- Each transaction has a date, value, product code, affiliated trade indicator, exporter country and firm ID, and importer firm ID.
- See also Besedes (2006); Bernard et al (2007); Blum et al, 2009a, 2009b; Albornoz et al, 2010; Carballo, Ottaviano and Martincus (2013).
Cohort survival

- Figures are averages over all 17 <1-yr-old cohorts, all 16 1-2 yr. old cohorts, etc.
- Most new Colombian exporters drop out of the U.S. market within a year.
But survival rates improve considerably after the first year.
The firms that survive their first year grow exceptionally rapidly (see also Ruhl and Willis, 2008).
Hence young cohorts typically maintain market share in first year or two, despite rapid attrition.

Post-1996 entrants account for about half of cumulative export expansion by 2005.
Buyer-Seller matches: durability

Most new matches fail within a year, but

- Chances of failure are higher for matches with small initial sales.
- Failure rates drop for all quantiles after the first year, but remain high.
- To sustain or increase exports, firms must continually replenish their foreign clientele.
Matches that start small tend to stay small.

After a match’s first year, there is no systematic tendency for its annual sales to grow.
A seriously Pareto client distribution

- Most firms have a single buyer, but the distribution of client counts across exporters is fat-tailed.
### Table 3: Transition Probabilities, Number of Clients

<table>
<thead>
<tr>
<th>t \ t+1</th>
<th>exit</th>
<th>texit</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6-10</th>
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<td>0.197</td>
<td>0.184</td>
<td>0.094</td>
<td>0.197</td>
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<td>6-10</td>
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<td>0.070</td>
<td>0.082</td>
<td>0.114</td>
<td>0.149</td>
<td>0.465</td>
<td>0.066</td>
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<td>11+</td>
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<td>0.000</td>
<td>0.000</td>
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<td>.</td>
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<td>0.440</td>
</tr>
</tbody>
</table>

Eaton et al. (\textsuperscript{a}Brown, \textsuperscript{b}NBER, \textsuperscript{c}U. de los An)
Firms engage in costly search to meet potential buyers at home and (possibly) abroad.

Firms new to the foreign market don’t know what fraction of buyers there will be willing to do business with them.

As they encounter potential buyers, firms gradually learn the scope of the market for their particular products, and they adjust their search intensities accordingly (learning).

Search costs fall as firms accumulate successful business relationships (network effects).

Maintaining a relationship with a buyer is costly, so a relationship that yields meager profits is dropped.

Three types of shocks: marketwide, firm-specific, match-specific
Three model components

1. A Seller-Buyer Relationship
2. Learning About Product Appeal from Encounters with Potential Buyers
3. Searching for Potential Buyers
Why continuous time?

- Two types of discrete events occur at random intervals, sometimes with high frequency
  - Sellers meet buyers
  - Once business relationships are established, orders are placed
- With continuous time formulation we can:
  - allow for an arbitrarily large number of events during any discrete interval
  - allow agents to update their behavior each time an event occurs
1. Relationship dynamics

profits from a shipment

- Define exogenous state variables:
  - $\phi_j$ productivity of seller $j$
  - $x_t^m$ size of market $m \in \{h, f\}$ (Markov jump process)
  - $y_{ijt}^m$ idiosyncratic shock to operating profits from shipment to buyer $i$ by seller $j$ in market $m$ (Markov jump process)
  - $\Pi^m$ profit function scalar (so that all exogenous state variables can be normalized to mean log zero)

- When buyer $i$ places an order with seller $j$ in market $m$ it generates operating profits:

\[ \pi(x_t^m, \phi_j, y_{ijt}^m) = \Pi^m x_t^m \phi_j^{\sigma - 1} y_{ijt}^m. \]

Superscripts and subscripts mostly suppressed hereafter:

\[ \pi_{\phi}(x, y) = \Pi x \phi^{\eta - 1} y \]
1. Relationship dynamics

value of a business relationship

- In active business relationships, buyers place orders with exogenous hazard $\lambda^b$. ▶ Details

- After each order, sellers must pay fixed cost $F$ to keep a business relationship active.

- Value to a type-$\phi$ seller of a relationship in state $\{x, y\}$:

$$\hat{\pi}_\phi(x, y) = \pi_\phi(x, y) + \max \{\hat{\pi}_\phi(x, y) - F, 0\}$$

- $\hat{\pi}_\phi(x, y)$ is continuation value to a type-$\phi$ seller of a relationship in state $\{x, y\}$. ▶ Details

- Continuation values depend negatively on
  - $\delta$: exogenous hazard of relationship death.
  - $\rho$: seller’s discount rate.
1. Relationship dynamics
expected value of a new relationship

- Sellers don’t know what value their next business relationship will begin from.
- Let \( Pr(y^s) \) be the probability of initial shock \( y^s \) determined by the ergodic distribution of \( y \).
- Expected value of a successful new encounter:

\[
\tilde{\pi}_\varphi(x) = \sum_{y^s} Pr(y^s) \tilde{\pi}_\varphi(s, y)
\]
2. Learning about product appeal
the "true" scope of the market

- Fraction of potential buyers in market $m$ who are interested in seller $j$’s product: $\theta_j^m \in [0, 1]$ of total potential buyers.
- Assume $\theta_j^m$’s are time-invariant, mutually independent draws from a beta distribution:

$$ r(\theta|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1 - \theta)^{\beta-1}, $$

- Expected value:

$$ E(\theta|\alpha, \beta) = \frac{\alpha}{\alpha + \beta}. $$

- Posterior beliefs, after meeting $n^m$ potential clients in market $m$, $a^m$ of whom want to do business:

$$ \overline{\theta}^m (a^m, n^m) = E [\theta^m|a^m, n^m] = \frac{a^m + \alpha}{n^m + \alpha + \beta} $$
3. Searching for buyers

the cost of search

- Seller continuously chooses the hazard $s$ with which she encounters a potential buyer at a flow cost $c(s, a)$
  - Maintain web site
  - Pay to be near top of web search listings
  - Attend trade fairs
  - Research foreign buyers
  - Send sales reps. to foreign markets
  - Maintain foreign sales office

- The number of successful encounters, $a$, allows for network effects (NYT 2/27/12: Panjiva, ImportGenius).

- Functional form used for estimation (Arkolakis, 2010):

$$c(s, a) = \kappa_0 \frac{(1 + s)^{(1+1/\kappa_1)} - 1}{(1 + a)^{\gamma \cdot (1+1/\kappa_1)} (1 + 1/\kappa_1)}$$
3. Searching for buyers
the value of search abroad

- Let \( V_\varphi(a, n, x) \) be the value of continued search for a type-\( \varphi \) firm with \( a \) successes in \( n \) meetings.

- The first-order for optimal search abroad is:

\[
\begin{align*}
    c_s(s^*, a) &= \bar{\theta}_{a,n}(\tilde{\pi}_\varphi(x) + V_\varphi(a + 1, n + 1, x)) \\
    &+ (1 - \bar{\theta}_{a,n}) V_\varphi(a, n + 1, x) - V_\varphi(a, n, x).
\end{align*}
\]

- Two pay-offs to a match:
  - profit stream (if the match is successful)
  - information on the scope of the market for your particular product.

- Learning opportunities assumed to be exhausted in the domestic (Colombian) market, so that \( c_s(s^*, a) = \theta_j \tilde{\pi}_\varphi(x) \).
Unidentified preference parameters taken from literature: $\rho = 0.05$, $\sigma = 5$

Assume $x^{f}$, $x^{h}$, and $y$ follow independent Markov jump processes.

- Estimate the $x^{f}$ and $x^{h}$ processes using real spending on manufactured goods in U.S. and Colombia, respectively, both expressed in real pesos.

Remaining parameters identified using indirect inference.

$$\Lambda = \left( \Pi^{h}, \Pi^{f}, \delta, F, \alpha, \beta, \sigma_{\varphi}, \lambda_{y}, \lambda_{b}, \gamma, \kappa_{0}, \kappa_{1} \right)$$
Indirect inference

moments

- **Profit scaling constants**, \((\Pi^h, \Pi^f)\)
  - means of log home and foreign sales
- **Shipment hazards** \((\lambda^b)\)
  - average annual shipment rates per match
- **Product appeal parameters** \((\alpha, \beta)\)
  - distribution of home and foreign sales
- **Firm productivity dispersion** \((\sigma_\varphi)\)
  - distribution of home and foreign sales
  - covariance of home and foreign sales
- **Search cost parameters** \((\kappa_0, \kappa_1, \gamma)\)
  - match rates
  - client frequency distribution (especially fatness of tail)
  - client transition probabilities
  - fraction of firms that export
Indirect inference
moments

- **Idiyncratic shocks to importers** ($\lambda^y$)
  - cross-plant variances in home and foreign sales
  - covariation of home and foreign sales
  - autocorrelation, match-specific sales
  - client frequency distribution, client transition probabilities

- **Match maintenance costs** ($F$)
  - client frequency distribution, client transition probabilities
  - sales among new versus established matches
  - age-specific match failure rates

- **Exogenous match separation hazard** ($\delta$)
  - separation rates after first year
  - age-specific match failure rates
  - client frequency distribution
### Transition probs., no. clients \((n^c)\)

| \(P[n_{jt+1}^c = 0 | n_{jt}^c = 1]\) | \(P[n_{jt+1}^c = 1 | n_{jt}^c = 1]\) | \(P[n_{jt+1}^c = 2 | n_{jt}^c = 1]\) | \(P[n_{jt+1}^c \geq 3 | n_{jt}^c = 1]\) | \(P[n_{jt+1}^c = 0 | n_{jt}^c = 2]\) | \(P[n_{jt+1}^c = 1 | n_{jt}^c = 2]\) | \(P[n_{jt+1}^c = 2 | n_{jt}^c = 2]\) | \(P[n_{jt+1}^c \geq 3 | n_{jt}^c = 2]\) |
|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| 0.618 | 0.321 | 0.048 | 0.013 | 0.271 | 0.375 | 0.241 | 0.113 |

### Share of firms exporting

<table>
<thead>
<tr>
<th>(E(1_{X_{jt}^f &gt; 0}))</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.299</td>
<td>0.351</td>
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</tbody>
</table>

### Log foreign sales on log domestic sales

<table>
<thead>
<tr>
<th>(\beta_{hf}^{\hat{\beta}})</th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td>0.727</td>
<td>0.515</td>
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<table>
<thead>
<tr>
<th>(s\hat{e}(\epsilon_{hf}))</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.167</td>
<td>1.424</td>
<td></td>
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</table>
### Data versus simulated statistics

<table>
<thead>
<tr>
<th>Match death hazards</th>
<th>Data</th>
<th>Model</th>
<th>Exporter exit rate</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Death rate, $A_{ijt-1}^m = 0$</td>
<td>0.694</td>
<td>0.857</td>
<td>Exit rate, $A_{ijt-1}^m = 0$</td>
<td>0.709</td>
<td>0.748</td>
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<tr>
<td>Death rate, $A_{ijt-1}^m = 1$</td>
<td>0.515</td>
<td>0.329</td>
<td>Exit rate, $A_{ijt-1}^m = 1$</td>
<td>0.383</td>
<td>0.099</td>
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<tr>
<td>Death rate, $A_{ijt-1}^m = 2$</td>
<td>0.450</td>
<td>0.304</td>
<td>Exit rate, $A_{ijt-1}^m = 2$</td>
<td>0.300</td>
<td>0.121</td>
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<tr>
<td>Death rate, $A_{ijt-1}^m = 3$</td>
<td>0.424</td>
<td>0.281</td>
<td>Exit rate, $A_{ijt-1}^m = 3$</td>
<td>0.263</td>
<td>0.055</td>
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<tr>
<td>Death rate, $A_{ijt-1}^m = 4$</td>
<td>0.389</td>
<td>0.305</td>
<td>Exit rate, $A_{ijt-1}^m = 4$</td>
<td>0.293</td>
<td>0.100</td>
</tr>
</tbody>
</table>

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## Data versus simulated statistics

<table>
<thead>
<tr>
<th>Log sales per client vs. no. clients</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_1^m$</td>
<td>2.677</td>
<td>0.842</td>
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<td>$\hat{\beta}_2^m$</td>
<td>-0.143</td>
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<tr>
<td>$se(\epsilon^m)$</td>
<td>2.180</td>
<td>1.622</td>
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<table>
<thead>
<tr>
<th>Ave. log sales by cohort age</th>
<th>Data</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>$\hat{E}(\ln X_{jt}^f</td>
<td>A_{jt}^c = 0)$</td>
<td>8.960</td>
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<tr>
<td>$\hat{E}(\ln X_{jt}^f</td>
<td>A_{jt}^c = 1)$</td>
<td>10.018</td>
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<td>$\hat{E}(\ln X_{jt}^f</td>
<td>A_{jt}^c = 2)$</td>
<td>10.231</td>
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<tr>
<td>$\hat{E}(\ln X_{jt}^f</td>
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<tr>
<td>$\hat{E}(\ln X_{jt}^f</td>
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<table>
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<tr>
<th>No. clients, inverse</th>
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<tr>
<td>$\hat{\beta}_1^c$</td>
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<td>$se(\epsilon^n^c)$</td>
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<td>0.128</td>
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</table>
## Data versus simulated statistics

<table>
<thead>
<tr>
<th>Match death prob regression</th>
<th>Data</th>
<th>Model</th>
<th>Log match sale autoreg.</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_0^d$</td>
<td>1.174</td>
<td>1.640</td>
<td>$\hat{\beta}_1^f$</td>
<td>0.811</td>
<td>0.613</td>
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<tr>
<td>$\hat{\beta}_d^d$</td>
<td>0.166</td>
<td>0.203</td>
<td>$\hat{\beta}_1^{\text{1st year}}$</td>
<td>0.233</td>
<td>0.370</td>
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<tr>
<td>$\hat{\beta}^{\text{1st year}}$</td>
<td>0.166</td>
<td>0.203</td>
<td>$s\hat{e}(\epsilon^f)$</td>
<td>1.124</td>
<td>0.503</td>
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<tr>
<td>$\hat{\beta}^d$</td>
<td>-0.070</td>
<td>-0.100</td>
<td>$\hat{\beta}_1^h$</td>
<td>0.976</td>
<td>0.896</td>
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<tr>
<td>$s\hat{e}(\epsilon^d)$</td>
<td>0.453</td>
<td>0.395</td>
<td>$s\hat{e}(\epsilon^h)$</td>
<td>0.462</td>
<td>0.683</td>
</tr>
</tbody>
</table>

### Match shipments per year

<table>
<thead>
<tr>
<th>$\hat{E}(n^s)$</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.824</td>
<td>3.770</td>
<td></td>
</tr>
</tbody>
</table>
fixed cost of maintaining a relationship: \( \exp(7.957) = 2,855 \), about 35\% of the value of a typical shipment.

only about \( \frac{\alpha}{(\alpha + \beta)} = 0.18 \) of the potential buyers a typical exporter meets are interested in doing business

success rates vary across exporters with standard deviation

\[
\sqrt{\frac{\alpha \beta}{((\alpha + \beta)^2(\alpha + \beta + 1))}} = 0.176
\]
Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand shock jump hazard</td>
<td>( \lambda_y )</td>
<td>0.532</td>
</tr>
<tr>
<td>demand shock jump size</td>
<td>( \Delta y )</td>
<td>0.087</td>
</tr>
<tr>
<td>shipment order arrival hazard</td>
<td>( \lambda_b )</td>
<td>8.836</td>
</tr>
<tr>
<td>std. deviation, log firm type</td>
<td>( \sigma_\phi )</td>
<td>0.650</td>
</tr>
<tr>
<td>network effect parameter</td>
<td>( \gamma )</td>
<td>0.298</td>
</tr>
<tr>
<td>search cost function curvature parameter</td>
<td>( \kappa_1 )</td>
<td>0.087</td>
</tr>
<tr>
<td>search cost function scale parameter</td>
<td>( \kappa_0 )</td>
<td>111.499</td>
</tr>
</tbody>
</table>

- Convexity of search cost function is important
- Cost of search at hazard \( s = 1 \): $5,786 when \( a = 0 \); $437 when \( a = 1 \).
- Cost of search at hazard \( s = 5 \): $5.277 \times 10^9$ when \( a = 0 \); $6,301$ when \( a = 20 \).
- "Lock-in" effect
Cohort selection and growth

- Cumulative sales: first 3 years (red), years 3-6 (green), years 6-9 (blue)
Isolating learning and network effects
restricted models

<table>
<thead>
<tr>
<th></th>
<th>benchmark</th>
<th>no learning</th>
<th>no network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Lambda )</td>
<td>( \Lambda^{NL} )</td>
<td>( \Lambda^{NN} )</td>
</tr>
<tr>
<td>exog. separation rate</td>
<td>( \delta )</td>
<td>0.267</td>
<td>0.516</td>
</tr>
<tr>
<td>fixed cost</td>
<td>( F )</td>
<td>7.957</td>
<td>10.238</td>
</tr>
<tr>
<td>( \theta ) dist. parameter</td>
<td>( \alpha )</td>
<td>0.716</td>
<td>0.512</td>
</tr>
<tr>
<td>( \theta ) dist. parameter</td>
<td>( \beta )</td>
<td>3.161</td>
<td>0.351</td>
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<tr>
<td>dmd. jump hazard</td>
<td>( \lambda_y )</td>
<td>0.532</td>
<td>0.713</td>
</tr>
<tr>
<td>dmd. jump size</td>
<td>( \Delta y )</td>
<td>0.087</td>
<td>0.060</td>
</tr>
<tr>
<td>order arrival hazard</td>
<td>( \lambda_b )</td>
<td>8.836</td>
<td>10.028</td>
</tr>
<tr>
<td>std. dev., log firm type</td>
<td>( \sigma_{\varphi} )</td>
<td>0.650</td>
<td>1.268</td>
</tr>
<tr>
<td>network effect</td>
<td>( \gamma )</td>
<td>0.298</td>
<td>0.112</td>
</tr>
<tr>
<td>fit metric</td>
<td>( D )</td>
<td>9.97 e+04</td>
<td>2.155 e+05</td>
</tr>
<tr>
<td>fit metric, no weighting</td>
<td>( \tilde{D} )</td>
<td>0.117</td>
<td>0.182</td>
</tr>
</tbody>
</table>

- no-learning model treats firms as knowing their exact \( \theta^f \) draws
- no-network model shuts down reputation effects by imposing \( \gamma = 0 \)
- both alternatives fit much worse than the benchmark model.
Successes, failures, and search intensity

![Graph showing search intensity vs. successes with different failure rates](image-url)
The policy function without network effects
A 20% reduction in search costs
A 20% reduction in fixed costs
A 20% increase in foreign market size
Micro patterns of transactions and buyer-seller relationships through the lens of the model:

- Large volume of small scale exporters explained by large volume of inexperienced firms, searching at a low level.
- High exit rate reflects short lifespan of typical match, combined with low-level search and learning about product appeal.
- Small number of major exporters reflects combination of skewed distribution of product appeal and reputation effects.

Search costs, multi-period matches, learning, and reputation effects combine to provide an explanation for hysteresis in trade.

- Reputation effects appear to be particularly important; may create insider clubs.
- Since learning is mainly relevant for new, marginal players, probably doesn’t have a big effect on short-run export dynamics.
A Digression: hazards

- From the perspective of time 0, let the probability that an event will occur before time $t$ be described by the exponential distribution:

$$ F[t] = 1 - e^{-qt} $$

- The likelihood of the event happening exactly at $t$ (the "hazard rate" at $t$) is then:

$$ \frac{f(t)}{1 - F(t)} = \frac{qe^{-qt}}{e^{-qt}} = q $$

- This hazard rate doesn't depend upon $t$. 
Suppose $k$ independent events occur with hazard $q_1, q_2, \ldots q_k$. The probability that none occur before $t$ is:

$$\prod_{j=1}^{k} (1 - F_j(t)) = e^{-t\sum q_j}$$

So by time $t$, at least one event occurs with probability $1 - e^{-t\sum q_j}$, and the likelihood that this happens exactly at $t$ is

$$\sum_j q_j \left[ e^{-t\sum q_j} \right] = \sum_j q_j$$
$x$ (market-wide) follows Markov jump process, hazard $q_{xx'}^X$ of transiting from state $x$ to state $x'$.  
$y$ (match-specific) follows Markov jump process, hazard $q_{yy'}^Y$ of transiting from state $y$ to state $y'$.  
$\lambda^X_x = \sum_{x' \neq x} q_{xx'}^X$ is hazard of a change in market-wide state $x$.  
$\lambda^Y_y = \sum_{y' \neq y} q_{yy'}^Y$ is hazard of a change in match-specific state $y$.  
$\lambda^b$ is hazard of a new purchase order from existing client.  
$\tau^b$ time until the next change in state, which occurs with hazard $\lambda^b + \lambda^X_x + \lambda^Y_y$.  

Eaton et al. (\textsuperscript{a}Brown, \textsuperscript{b}NBER, \textsuperscript{c}U. de los Andes, \textsuperscript{d}U. Copenhagen, \textsuperscript{e}Census Bureau (CES), \textsuperscript{f}Penn State)
Relationship dynamics

the continuation value

- $\delta$ exogenous hazard of relationship death.
- $\rho$ seller’s discount rate.

Continuation value of a business relationship in state $(x, y)$ for a type-$\varphi$ exporter:

$$\hat{\pi}_\varphi(x, y) = \mathbb{E}_{\tau_b} \left[ e^{-(\rho+\delta)\tau_b} \frac{1}{\lambda^b + \lambda_x^X + \lambda_y^Y} \right]$$

$$\cdot \left( \sum_{x' \neq x} q_{xx'}^X \hat{\pi}_\varphi(x', y) + \sum_{y' \neq y} q_{yy'}^Y \hat{\pi}_\varphi(x, y') + \lambda^b \hat{\pi}_\varphi(x, y) \right)$$

$$= \frac{1}{h} \left( \sum_{x' \neq x} q_{xx'}^X \hat{\pi}_\varphi(x', y) + \sum_{y' \neq y} q_{yy'}^Y \hat{\pi}_\varphi(x, y') + \lambda^b \hat{\pi}_\varphi(x, y) \right)$$

where

$$h = \rho + \delta + \lambda^b + \lambda_x^X + \lambda_y^Y$$
Learning about product appeal
experience and expected success rates

- Suppress market superscripts to reduce clutter.
- The **prior distribution** is:

\[
r(\theta|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} (\theta)^{\alpha-1} (1 - \theta)^{\beta-1},
\]

- **The likelihood**: Given \(\theta\), and given that a seller has met \(n\) potential buyers, the probability that \(a\) of these buyers were willing to buy her product is binomially distributed:

\[
q[a|n, \theta] = \binom{n}{a} [\theta]^a [1 - \theta]^m]^{n-a}.
\]
Learning about product appeal
experience and expected success rates

- **The posterior** distribution for $\theta$:
  \[ p(\theta|a, n) \propto q[a|n, \theta] \cdot r(\theta|\alpha, \beta) \]

- The expected success rate after $a$ successes in $n$ trials is thus:
  \[ \bar{\theta}(a, n) = E[\theta|a, n] = \frac{a + \alpha}{n + \alpha + \beta} \]

- Sellers base their search intensity on this posterior mean.
Searching for buyers
the value of search

The value of continued search for a type-$\varphi$ firm with $a$ successes in $n$ meetings is:

$$V_{\varphi}(a, n, x) = \max_s \mathbb{E}_{\tau_s} \left[ -c(s, a) \int_0^{\tau_s} e^{-\rho t} dt + \frac{e^{-\rho \tau_s}}{s + \lambda^X_x} \cdot \left( \sum_{x' \neq x} q^X_{xx'} V_{\varphi}(a, n, x') ight) ight]$$

$$+ s \left[ \bar{\theta}_{a,n}(\tilde{\pi}_\varphi(x) + V_{\varphi}(a + 1, n + 1, x) + (1 - \bar{\theta}_{a,n}) V_{\varphi}(a, n + 1, x) \right]$$

where:

- $\lambda^X_x = \sum_{x' \neq x} q^X_{xx'}$ is the hazard of any change in the market-wide state $x$.
- $\tau_s$ is the random time until the next search event, which occurs with hazard $s + \lambda^X_x$. 

Eaton et al. (\textsuperscript{a}Brown, \textsuperscript{b}NBER, \textsuperscript{c}U. de los An) Search and Export Dynamics 11/14/14 46 / 54
Searching for buyers
the value of search

Taking expectations over $\tau_s$ yields:

$$V_\phi(a, n, x) = \max_s \frac{1}{\rho + s + \lambda_x} \left[ -c(s, a) + \sum_{x' \neq x} q_{xx'}^X V_\phi(a, n, x') \right]$$

$$+ s \left\{ \bar{\theta}_{a,n} \left[ \tilde{\pi}_\phi(x) + V_\phi(a + 1, n + 1, x) \right] + (1 - \bar{\theta}_{a,n}) V_\phi(a, n + 1, x) \right\}$$

The first-order condition is thus:

$$c_s(s^*, a) = \bar{\theta}_{a,n} \left( \tilde{\pi}_\phi(x) + V_\phi(a + 1, n + 1, x) \right)$$

$$+ (1 - \bar{\theta}_{a,n}) V_\phi(a, n + 1, x) - V_\phi(a, n, x).$$
Searching for buyers
when the truth is known: the domestic market

- In the domestic market the reward to search depends on $a$ and $n$ only through network effects.
- The value of search at home is thus simply:

$$V_\phi(x) = \max_s \frac{1}{\rho + s + \lambda_x} \left[ -c(s, a) + \sum_{x' \neq x} q_x^{x'} V_\phi(x') + s\theta_j \bar{\pi}_\phi(x) \right]$$

- The associated first-order condition is:

$$c_s(s^*, a) = \theta_j \bar{\pi}_\phi(x).$$
Assume $x^f$, $x^h$, and $y$ follow independent Ehrenfest diffusion processes.

Any variable $z$ that obeys this process is discretized into $2e + 1$ possible values, $e \in I^+$: $z \in \{-e\Delta, -(e-1)\Delta, \ldots, 0, \ldots, (e-1)\Delta, e\Delta\}$.

Process jumps with hazard $\lambda_z$, and when it does so:

$$z' = \begin{cases} 
  z + \Delta & \text{with probability } \frac{1}{2} \left(1 - \frac{z}{e\Delta}\right) \\
  z - \Delta & \text{with probability } \frac{1}{2} \left(1 + \frac{z}{e\Delta}\right) \\
  \text{other} & 0 
\end{cases}$$

As the grid becomes finer, this type of random variable asymptotes to an Ornstein-Uhlenbeck process:

$$dz = -\mu z dt + \sigma dW$$
Estimation

The exogenous state variables

- If \( z \) observed at regular intervals, can estimate \( \mu \) and \( \sigma \) by regressing \( z \) on lagged \( z \)
- For \( x^f, x^h \), obtain maximum likelihood estimates of \( \mu \) and \( \sigma \) using logged and de-meaned time series on total real consumption of manufactured goods in each country.
- Recover \( \lambda_z \) and \( \Delta \) using Shimer’s result that asymptotically, 
  \[
  \mu = \lambda_z / e, \quad \sigma = \sqrt{\lambda_z} \Delta.
  \]
- This gives us the \( q_{xx}^X \) values needed to construct \( q_{yy}^Y \)'s for home and foreign markets.
- Since \( y \) is unobservable, recover the parameters of its jump processes using the structure of the dynamic model.
### Estimation

The exogenous state variables

<table>
<thead>
<tr>
<th>Market-wide Shock Processes ( (x^f, x^h) )</th>
<th>Colombia</th>
<th>United States</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Orstein-Uhlenbeck Parameters</strong></td>
<td></td>
<td></td>
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<tr>
<td>( \mu ) Mean Reversion</td>
<td>0.171</td>
<td>0.174</td>
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<tr>
<td>( \sigma ) Dispersion</td>
<td>0.003</td>
<td>0.058</td>
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<tr>
<td><strong>Ehrenfest Process Parameters</strong></td>
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<td></td>
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<tr>
<td>( \lambda ) Jump Hazard</td>
<td>1.200</td>
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<tr>
<td>( \Delta ) Jump Size</td>
<td>0.003</td>
<td>0.053</td>
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<tr>
<td>grid points</td>
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</table>

Eaton et al. (\textsuperscript{a}Brown, \textsuperscript{b}NBER, \textsuperscript{c}U. de los Andes, \textsuperscript{d}U. Copenhagen, \textsuperscript{e}Census Bureau (CES), \textsuperscript{f}Penn State)
Indirect inference (Gouriéroux and Monfort, 1996)

- Using reduced-form auxiliary regressions and/or moments, summarize key relationships in the data using a vector of statistics ($\hat{\mathbf{M}}$).
- For a candidate set of parameter values ($\Lambda$), simulate same statistics using the model $\hat{\mathbf{M}}^s(\Lambda)$.
- Construct the loss function:

$$Q(\Lambda) = \left( \hat{\mathbf{M}} - \hat{\mathbf{M}}^s(\Lambda) \right)' \Omega \left( \hat{\mathbf{M}} - \hat{\mathbf{M}}^s(\Lambda) \right)$$

where $\Omega$ is a positive definite weighting matrix.
- Use a robust algorithm to search parameter space for $\hat{\Lambda} = \arg \min Q(\Lambda)$.
Parameters
The no-learning model

- Rapid turnover of novice exporters less likely:
  - discourages inexperienced low-\(\theta^f\) firms from exploring foreign markets
  - eliminates learning-based exit.

- High-\(\theta^f\) firms do not intensify their search efforts as they receive positive feedback.

- Lower productivity firms induced to participate in export markets by a
  - rightward shift in \(\theta^f\) distribution and
  - higher values for \(\Pi^f\) and \(\lambda_b\)

- Match failure rates and market exit rates are sustained by
  - higher values for \(F, \delta,\) and \(\lambda_y\).

- Model badly overstates the share of firms that export, overstates the relationship between sales per client and number of clients.
Parameters

The no-network model

- Model moves part way toward matching the Pareto shape by reducing the convexity of the search cost function, $\kappa_1$.
- This is an imperfect fix because all exporters are equally affected by $\kappa_1$, not just the larger ones.
- Various other adjustments occur, including:
  - modest increase in $F$,
  - rightward shift in the $\theta$ distribution, an
  - increase in the variance of $\phi$,
  - increase in the jump hazard for buyer shocks, $\lambda_y$
- Client distribution is far from Pareto: model is unable to explain the existence of very large exporters; overstates the fraction of firms that export.